Fast and Trustworthy SMT Solving for String Constraints

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Aug 24, 2022

Overview

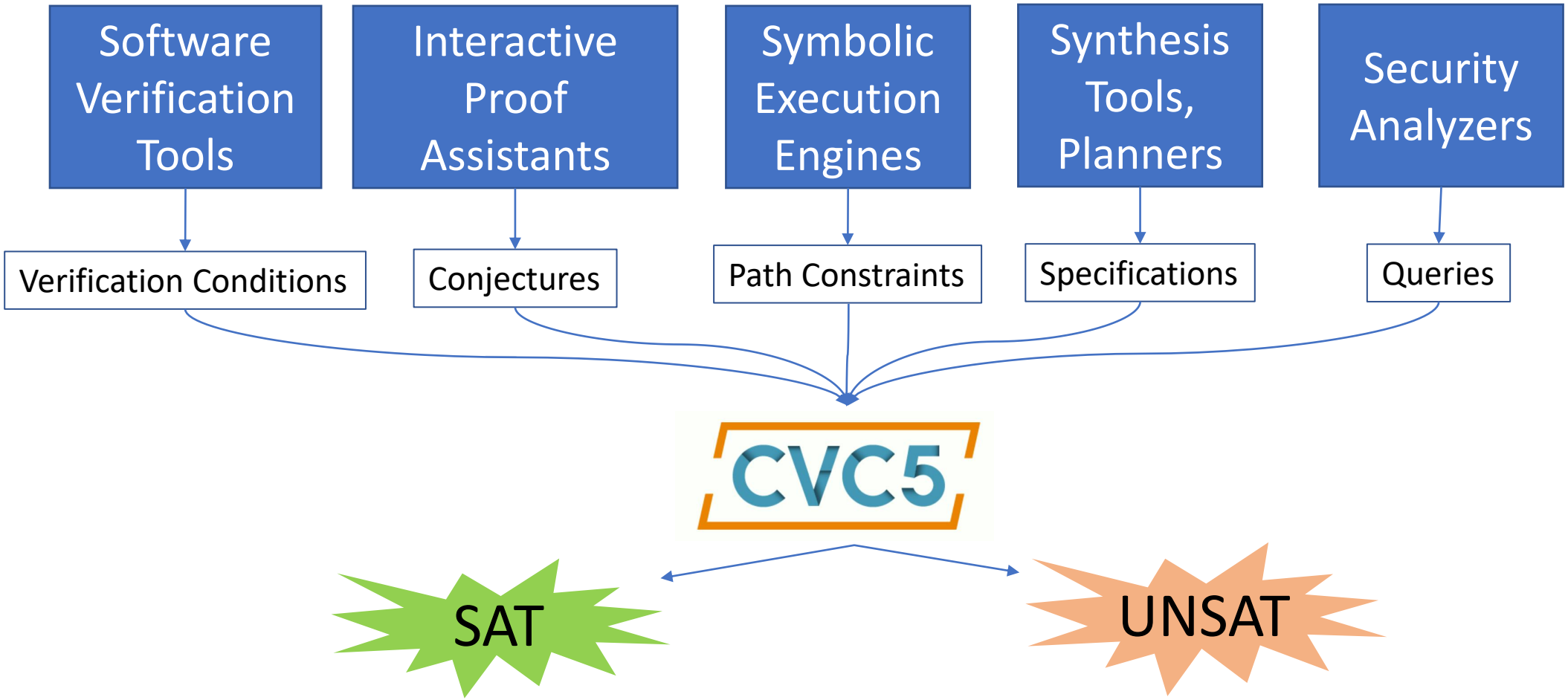
- Satisfiability Modulo Theories (SMT) solvers widely used tools

  $\Rightarrow$ the SMT solver cvc5

- cvc5 for strings and regular expressions
  - ***Fast:*** new advances in SMT string solving
  - ***Trustworthy:*** producing externally checkable proofs

- Future Directions

Satisfiability Modulo Theories (SMT) Solvers



⇒ SMT solvers are fully automated reasoners, widely used in applications

cvc5: A Versatile and Industrial-Strength SMT Solver*

Haniel Barbosa[3], Clark Barrett[1], Martin Brain[4], Gereon Kremer[1], Hanna Lachnitt[1], Makai Mann[1], Abdalrhman Mohamed[2], Mudathir Mohamed[2], Aina Niemetz[1(✉)], Andres Nötzli[1], Alex Ozdemir[1], Mathias Preiner[1], Andrew Reynolds[2], Ying Sheng[1], Cesare Tinelli[2], and Yoni Zohar[1,5]

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- cvc5 is latest SMT solver in CVC line of tools
  - Open source, builds on code base of CVC4
  - 1.0 launched in April 2022

⇒ Best tool paper, ETAPS 2022

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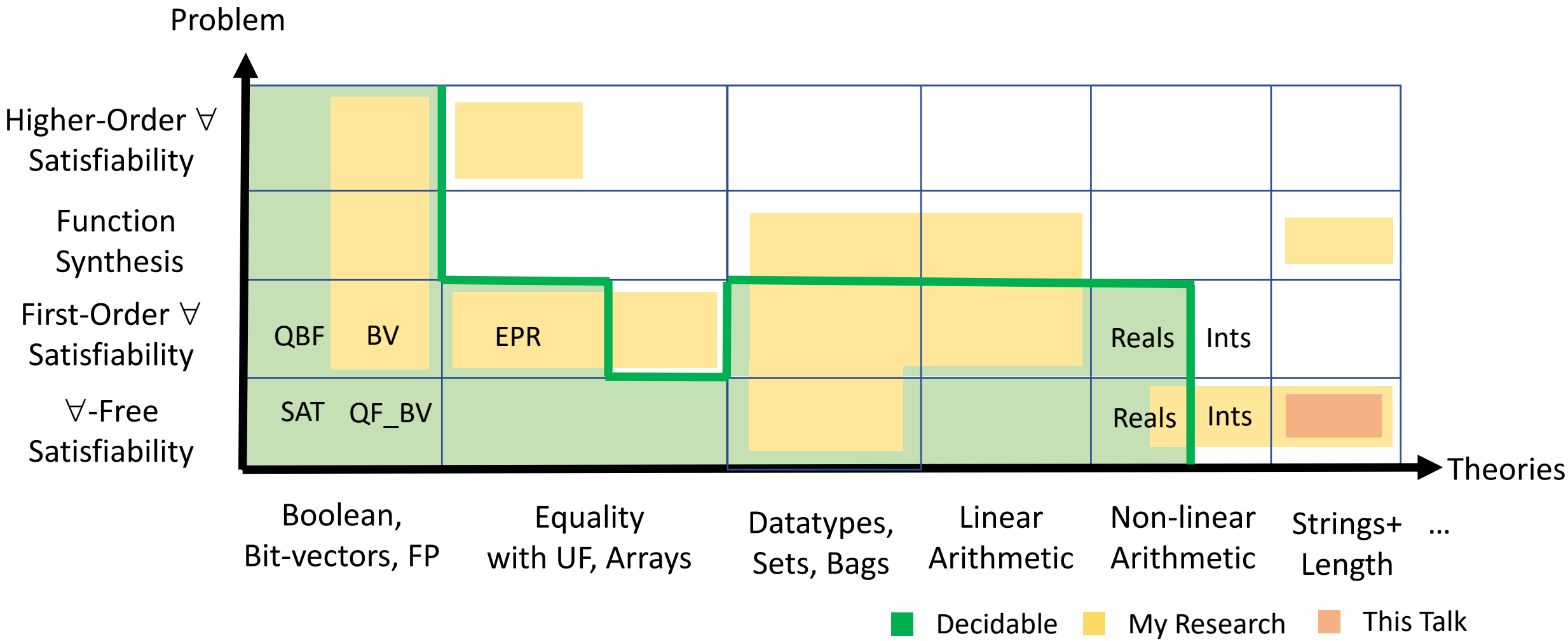
cvc5: A Versatile SMT Solver

- Support for many theories
  - Arithmetic, Bit-vectors, Arrays, Datatypes, Floating-Points, *Strings*
  - **Extended:** Sets, Multisets, Finite Fields

- Many features:
  - `get-model`, `get-unsat-core`, `get-proof`
  - **Extended:** syntax-guided synthesis, `get-interpolant`, `get-abduct`, `get-quant-elim`

$\Rightarrow$ If you have a new problem domain, we'd like to support it!

cvc5: SMT and beyond



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cvc5: An Industrial-Strength SMT Solver

- Efficient solving algorithms
  - Best overall performance* SMT-COMP 2018, 2019, 2020, 2021, 2022

    * shared with predecessor CVC4

- High coding standards
  - Streamlined API, high code coverage, code reviews

- Extensively tested
  - 3000+ hand-crafted regressions and counting
  - Fuzzed internally (Murxla **[Niemetz et al CAV22]**) and by external groups

- **New:** Produces externally checkable proofs

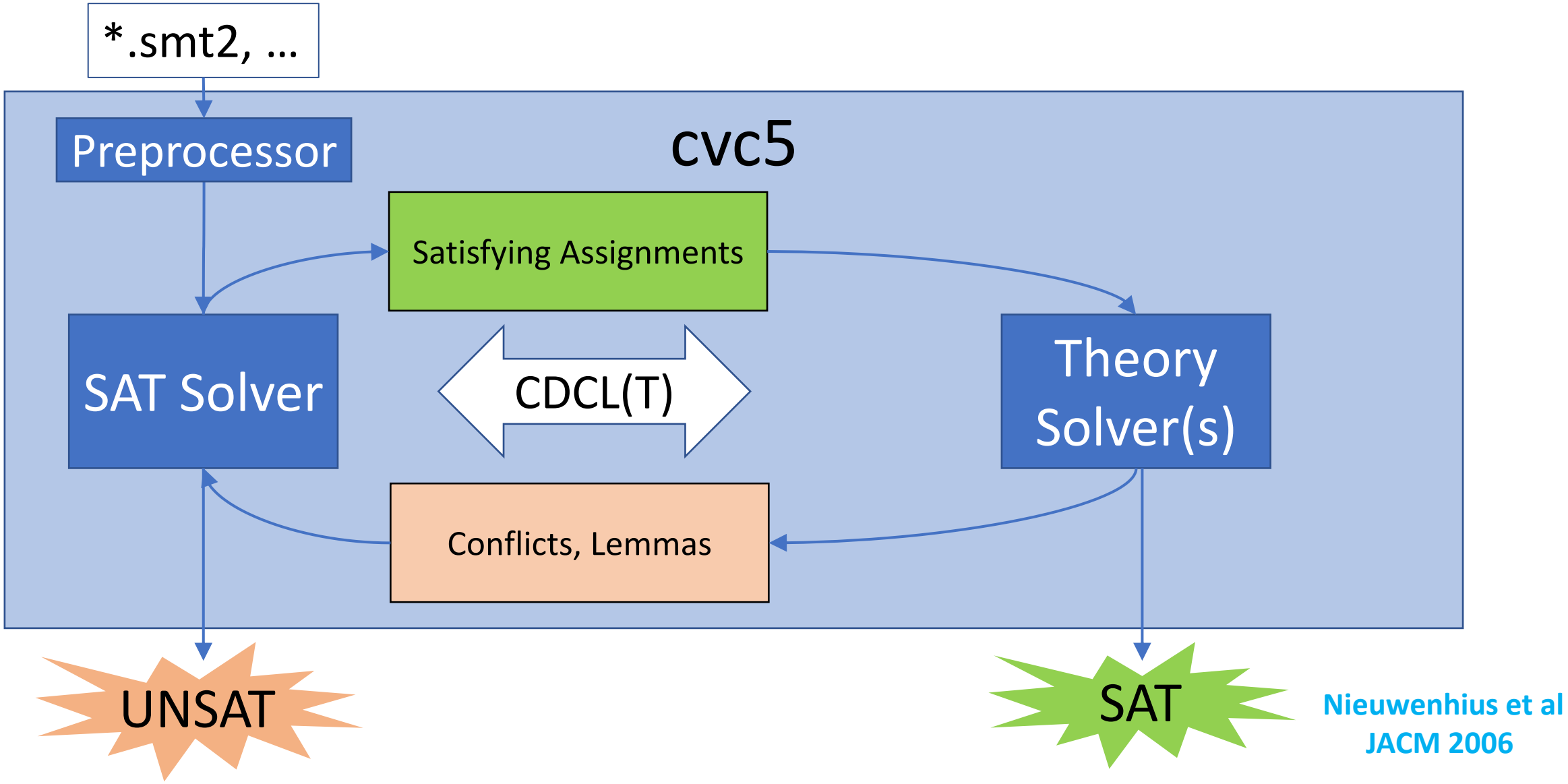
cvc5: state-of-the-art SMT solver *for Strings*

- **Fast** solving techniques
  - CDCL(T)
  - Core calculus for strings and length constraints
  - Extensions to extended functions and regular expressions

- **Trustworthy** results
  - Proof production for the full theory of strings

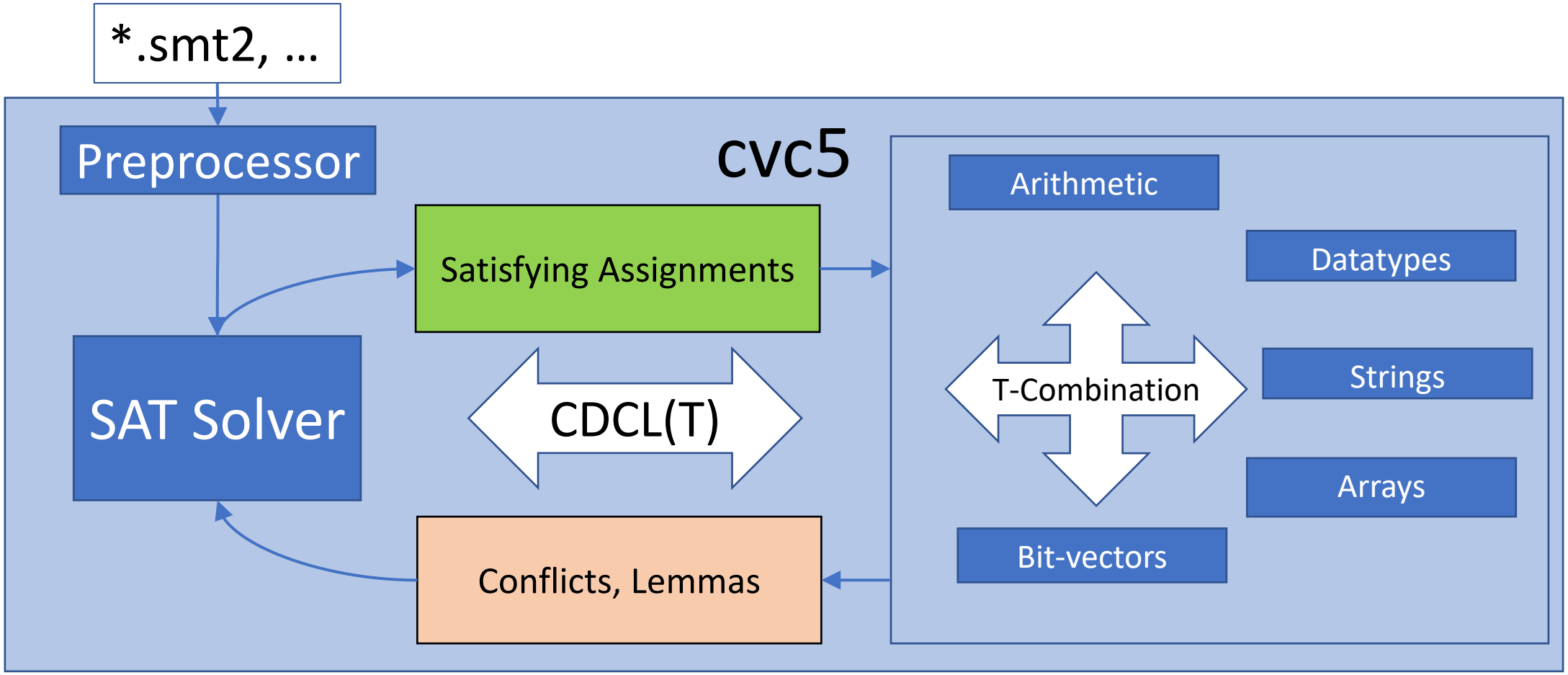
Designing a **Fast** String Solver

Architecture of cvc5



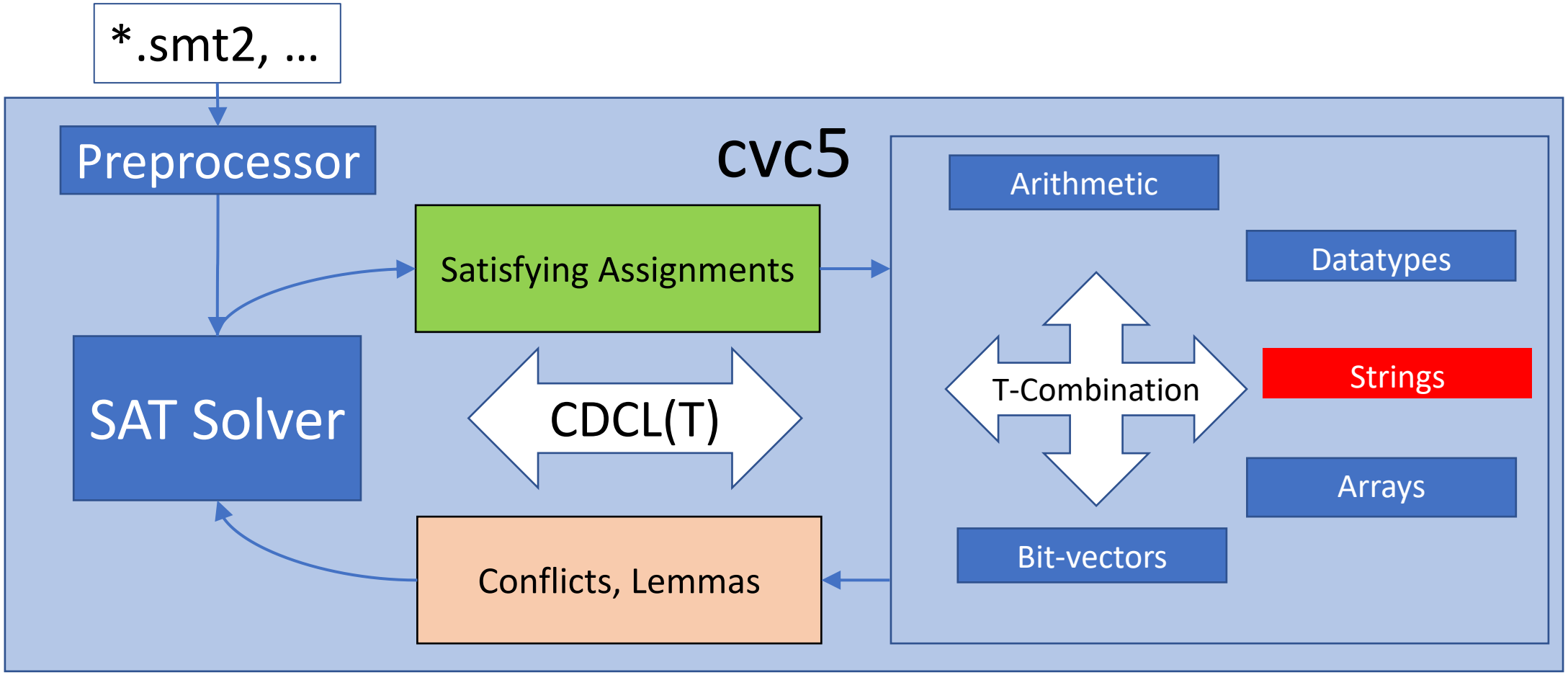
Nieuwenhius et al
JACM 2006

Architecture of cvc5



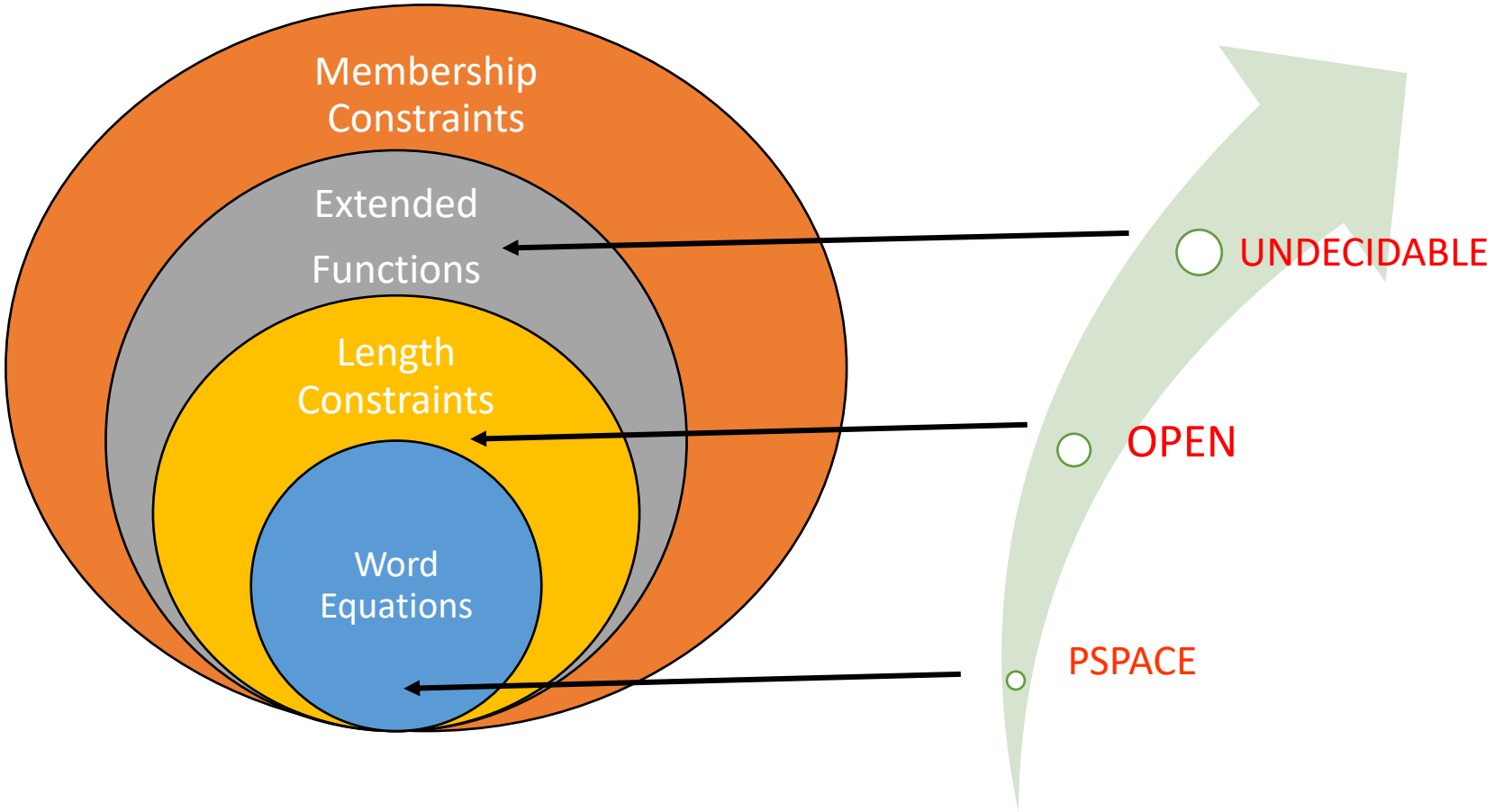
- Centralized methods (Nelson-Oppen, polite) for combining theories

Architecture of cvc5



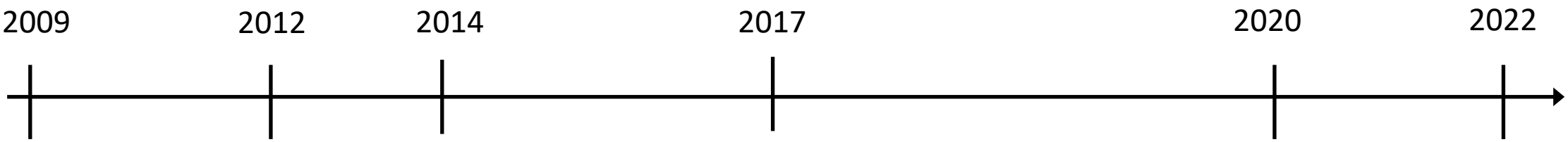
- Focus of this talk: solver for the *theory of strings and regular expressions*

Strings and RegExp: Theoretical Challenges



- Many applications require *extended string functions* and *RegEx memberships*
  - `ctn(x,"a"), to_lower(x)="abc", x∈range("A","Z")`

SMT Solvers for Strings: Timeline



CVC papers on strings

[Liang et al CAV 14]

[Reynolds et al CAV 17]

[Reynolds et al CAV 19]

[Reynolds et al FMCAD 20]

[Reynolds et al IJCAR 20]

[Noetzli et al CAV 22]

[Noetzli et al FMCAD 22]

Other SMT String Solvers

Z3Str

S3

Norn

Z3Str2

Z3Seq

Trau

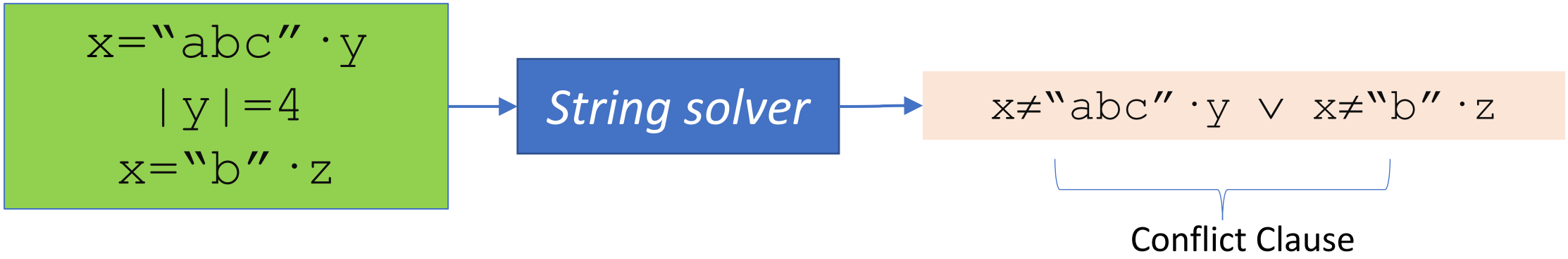
Z3Str3

Z3Str4

OSTRICH

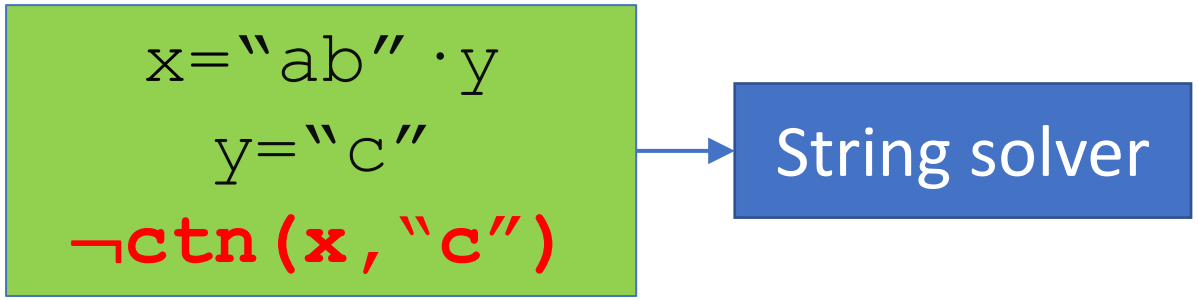
….

A Theory Solver for Strings [Liang et al, CAV 14]



- Designed a string solver for concat+length that is:
  - **Refutation and model sound** ("unsat" and "sat" can be trusted)
  - **Not terminating** in general
  - **Efficient** in practice

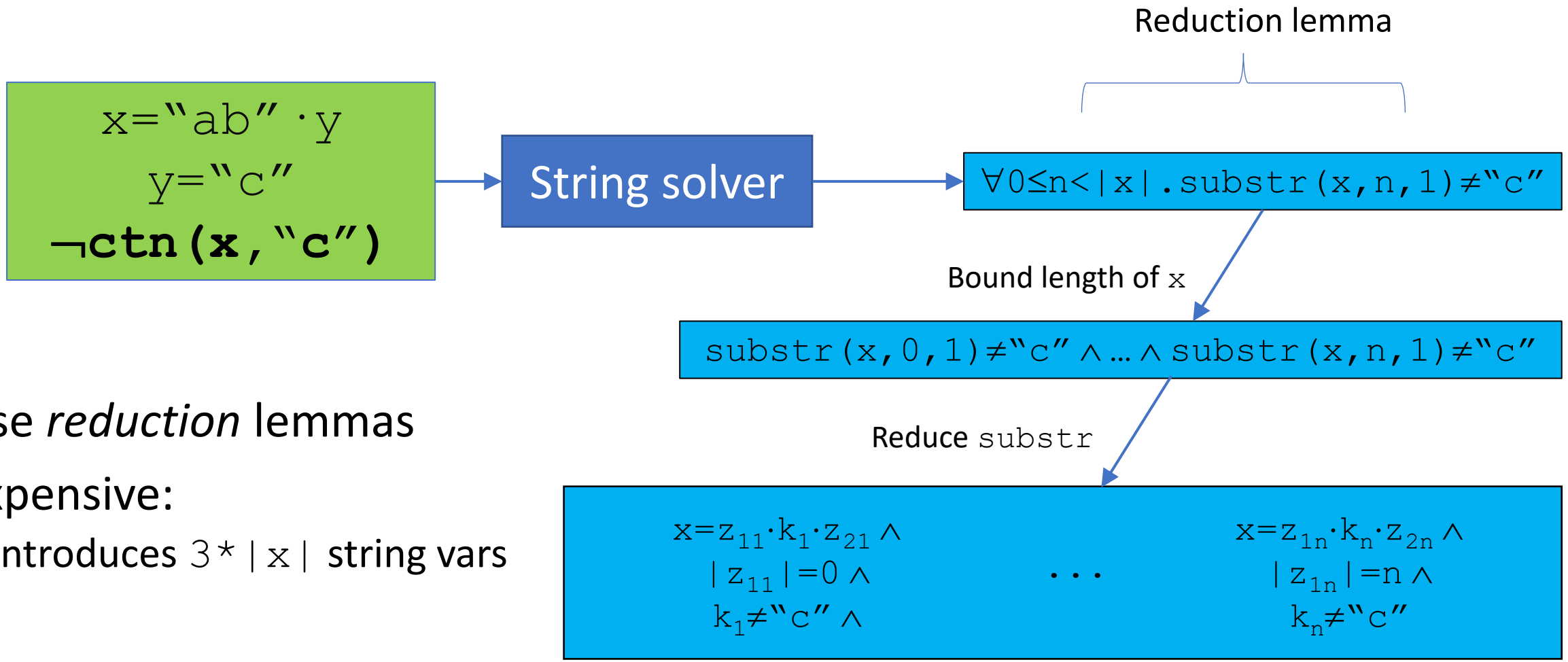
Extended Theory of Strings [Reynolds et al CAV17]



- Support *extended* string functions commonly used in applications
  - `substr(x,n,m)` substring of `x` at position `n` of length at most `m`
  - `ctn(x,y)` true if `x` contains substring `y`
  - `indexof(x,y,n)` position of `y` in `x` starting from position `n`
  - `replace(x,y,z)` result of replacing first occurrence of `y` in `x` by `z`

- For example: **`¬ctn(x,"c")`** denotes `x` does not contain substring `"c"`

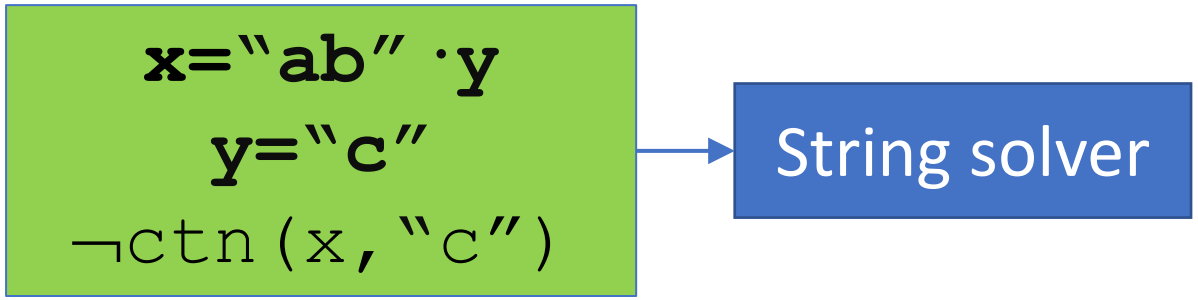
Extended Theory of Strings [Reynolds et al CAV17]



- Use *reduction* lemmas
- Expensive:

  Introduces 3*|x| string vars

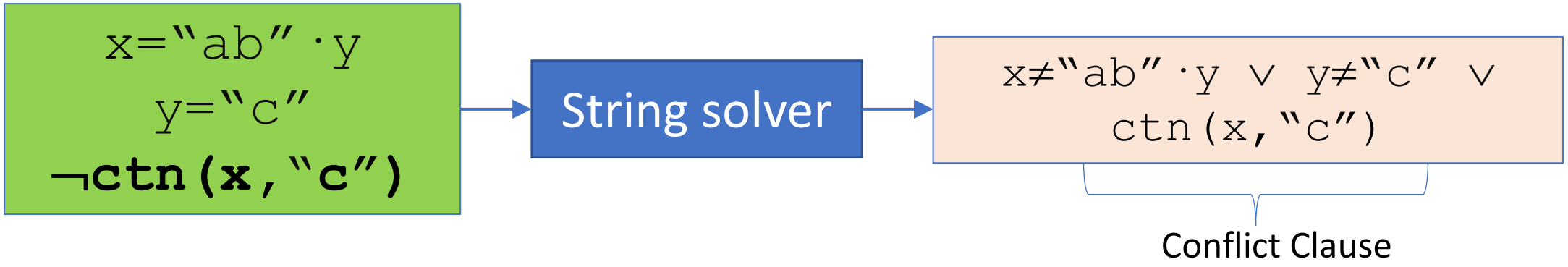
Context-Dependent Simplification [Reynolds et al CAV17]



- Alternatively: use *context-dependent* simplification:

$$x="ab"\cdot y \wedge y="c" \models x="abc"$$

Context-Dependent Simplification [Reynolds et al CAV17]



- Alternatively: use *context-dependent* simplification:

  $x = \text{"ab"} \cdot y \wedge y = \text{"c"} \models x = \text{"abc"}$

  - Thus:

    $\neg ctn(x, \text{"c"})\{x \rightarrow \text{"abc"}\} \Leftrightarrow \neg ctn(\text{"abc"}, \text{"c"}) \Leftrightarrow \bot$

    By substitution — By rewriting

Recent Developments for Theory of Strings

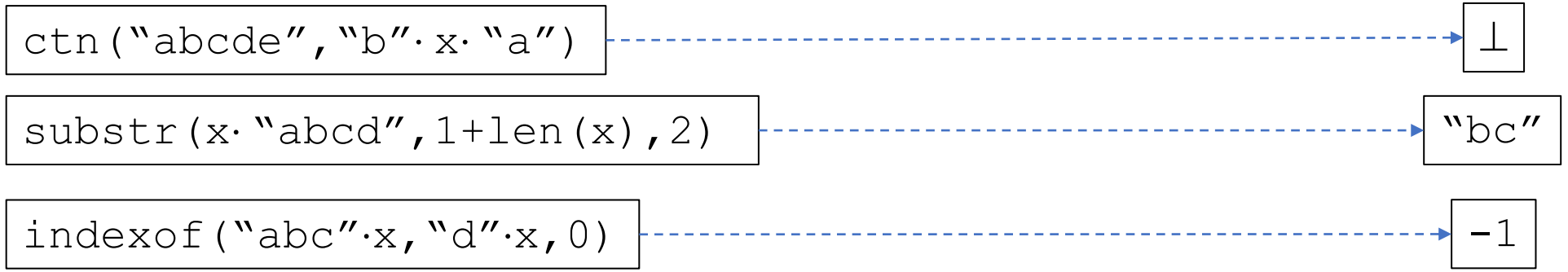
- Context-dependent simplifications
  - Use aggressive rewriting **[Reynolds et al CAV 2019]**
  - Applied eagerly **[Noetzli et al CAV 2022]**

- Reduction lemmas
  - Leverage String-to-code point (`code`) conversion **[Reynolds et al IJCAR 2020]**
  - Improved encodings **[Reynolds et al FMCAD 2020]**
  - Applied lazily based on model **[Noetzli et al CAV 2022]**

Rewriting based on High-Level Abstractions

- Unlike arithmetic:

…rewrite rules for strings are *highly non-trivial*:

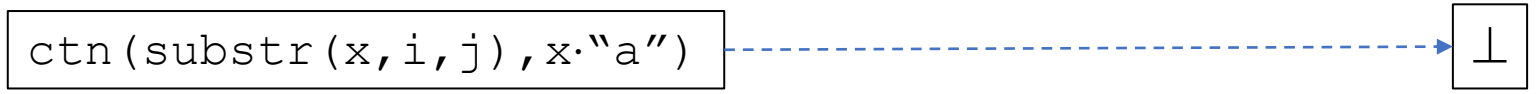


- Used syntax-guided synthesis to search for rewrite rules
  - Wrote 3000+ new LOC (C++) in cvc5's string rewriter module

Reynolds et al CAV19

Rewriting based on High-Level Abstractions

- Rules based on high-level abstractions
  - Strings as #characters (e.g. reasoning about their length):



…since the second argument is longer than the first

  - Strings as elements in containment lattice:

…since `x·y` contains `x`, which contains the second argument

  - Strings as multisets of characters:

…since LHS contains at least 1 more occurrences of `"a"`

⇒ With more rewrites, context-dependent simplification applies *more often*

A Decision Procedure for Code Points

- Even with aggressive simplification, still require reductions
  - Many extended function reductions require reasoning about characters

- **Idea**: extend core solver for strings to reason about *code points*
  - Assume ordering on characters of alphabet $\mathcal{A}$:
    - $c_1 < \ldots < c_{|\mathcal{A}|-1}$ where for each c_i, we call i its code point
  - $\texttt{code} : \texttt{Str} \rightarrow \texttt{Int}$ is interpreted as:
    1. For w in $\mathcal{A}^1$, $\texttt{code}(w)$ is the code point of the single character in w
    2. For all other w, $\texttt{code}(w)$ is -1

- Fragment with string length + string code point (w/o concatenation):
  - Procedure is **sound, complete, terminating**
  - Can be combined modularly with the existing string solver

Reynolds et al IJCAR20

A Decision Procedure for Code Points

- *More efficient reductions* that leverage `code`, including:

- Conversion between strings and integers `to_int(x)`:
  - ⊗ `ite(x[i]="9",9,ite(x[i]="8",8, … ite(x[i]="0",0,-1)…)`
  - ⇒ `ite(48≤`**`code`**`(x[i])≤57,`**`code`**`(x[i])-48, -1)`

…note `48` is Unicode for "`0`"

- Regular expression ranges $x \in range(c_1, c_2)$:
  - ⊗ $len(x)=1 \wedge (x=c_1 \vee \ldots \vee x=c_2)$
  - ⇒ **code**$(c_1) \leq$ **code**$(x) \leq$ **code**(c_2)

  - Similar for conversions to lower/upper case, lexicographic ordering

⇒ Reasoning about code points is deferred to cvc5's linear arithmetic solver

Revisiting Reductions for Strings

- **Observation:** there exist equivalent ways of expressing the same constraint
  - For strings `x`,`y`:

$$\exists z.\, x = z \cdot y \wedge \mathtt{len}(z) = 1$$
$$\mathtt{substr}(x, 1, \mathtt{len}(x) - 1) = y$$
$$x \in \Sigma \cdot \mathtt{to\_re}(y)$$

… `y` is the result of removing the first character from `x`

- **Idea:** *reuse variables* in extended functions and regular expression reductions

Reynolds et al FMCAD20

Revisiting Reductions for Strings

- Reduction for $\texttt{substr}(x,1,n)$:

  $\Rightarrow\ (\texttt{len}(x)>0 \wedge n>0) \Rightarrow (x=\mathbf{z_1}\cdot z_2\cdot z_3 \wedge \texttt{len}(\mathbf{z_1})=1 \wedge \texttt{len}(z_2)\leq n \wedge \ldots)$

- Map W from variables to "witness form"
  - E.g. $W(\mathbf{z_1})=\texttt{substr}(x,0,1)$, $W(z_2)=\texttt{substr}(x,1,n)$, $W(z_3)=\ldots$

- Reduction for $x\in\Sigma\cdot\texttt{to\_re}(y)$:

  $\otimes\ x=z_4\cdot z_5 \wedge z_4\in\Sigma \wedge z_5\in\texttt{to\_re}(y)$

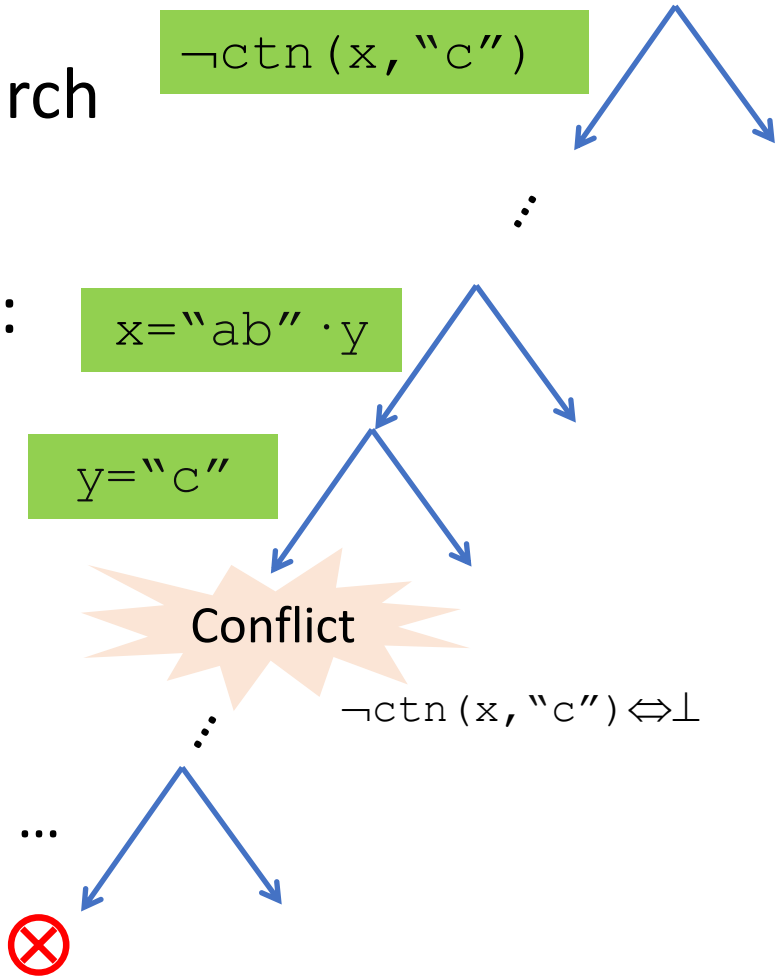
  $\Rightarrow\ x=\mathbf{z_1}\cdot z_5 \wedge \mathbf{z_1}\in\Sigma \wedge z_5\in\texttt{to\_re}(y)$

  … since first component z_4 also corresponds to $\texttt{substr}(x,0,1)$

- Witness forms can leverage rewriting $\downarrow$, share variables z_i and z_j when $W(z_i)\downarrow=W(z_j)\downarrow$

Even *Faster Conflicts* and Lazier Reductions

- **Idea:** apply simplifications eagerly during CDCL(T) search
- Instrument congruence closure to detect conflicts via:
  - Rewriting
  - Inferred properties of equivalence classes
    - Upper/lower bounds for integer equivalence classes
    - Prefix and suffix approximations for string equivalence classes
- Report conflicts as soon as they arise
  - Avoids redundant search space



Noetzli et al CAV22

Even Faster Conflicts and *Lazier Reductions*

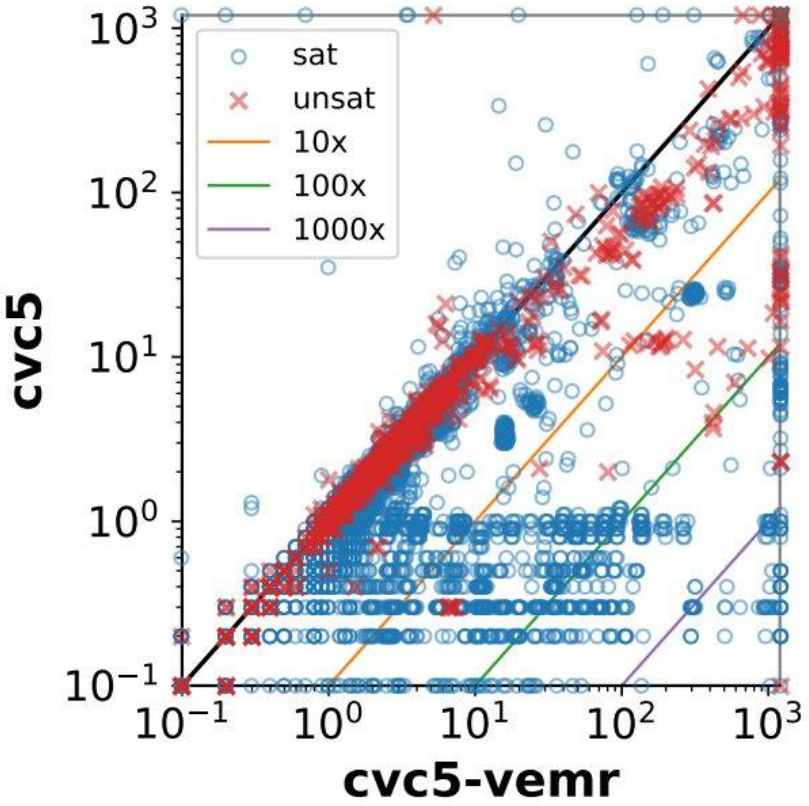
- Avoid reasoning about *unnecessary* reduction lemmas

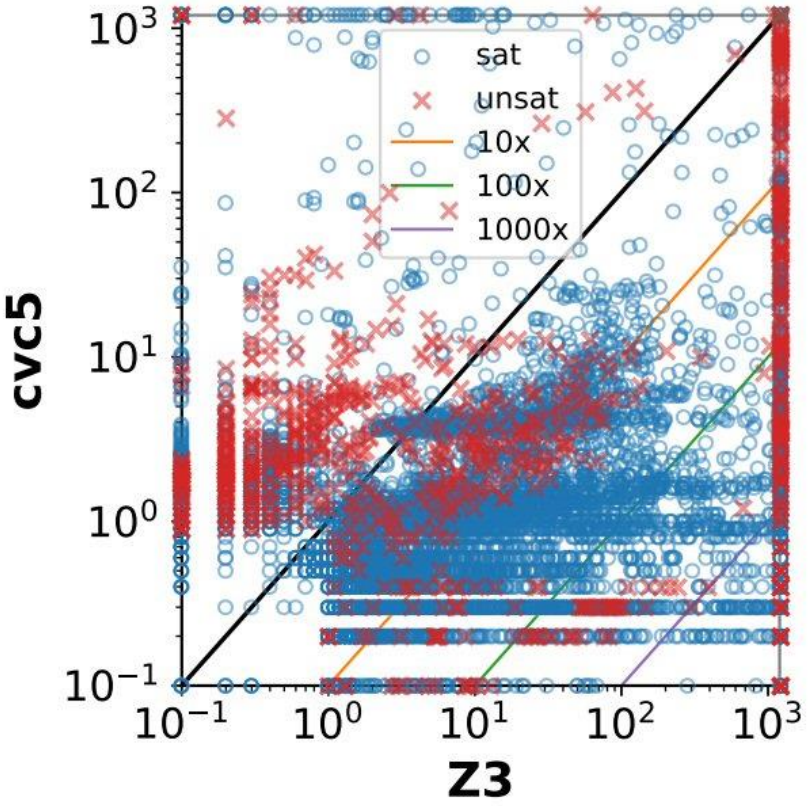
- Regular expression inclusion tests
  - ⊗ E.g. do not reduce $x \in \Sigma^* \cdot a \cdot \Sigma^*$ if already reduced $x \in \Sigma^* \cdot a \cdot \Sigma^* \cdot b \cdot \Sigma^*$
    - Since $L(\Sigma^* \cdot a \cdot \Sigma^* \cdot b \cdot \Sigma^*) \subseteq L(\Sigma^* \cdot a \cdot \Sigma^*)$
  - Fast incomplete procedure for language inclusion
  - Can also be used for finding conflicts

- Model-based reductions
  - Construct candidate model M
  - ⊗ Do not reduce e.g. string predicates p that are already satisfied by M
  - Often, *negative* reg exp memberships are satisfied by current model

Noetzli et al CAV22

Even Faster Conflicts and Lazier Reductions





- Results on 10857 SMT-LIB string benchmarks, 1200 second timeout
  - cvc5 solves 10347, z3 solves 8863

Noetzli et al CAV22

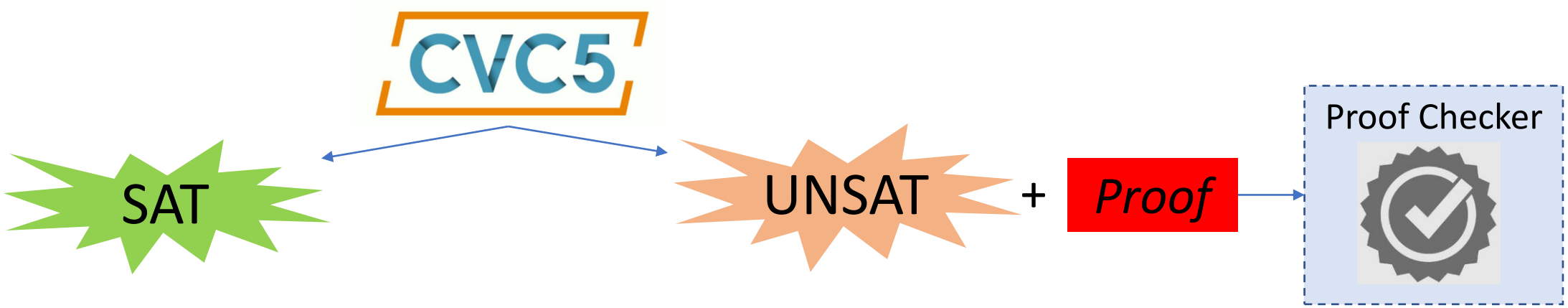
Designing a **Trustworthy** String Solver

The Need for SMT Proofs

- Correctness of cvc5 is highly critical to applications
  - In particular, refutational soundness

    ⇒ An incorrect UNSAT response may tell a user a system is safe when it is not!
- cvc5 is a highly complex code base
  - 150k+ LOC, constantly changing with new algorithmic advancements

    ⇒ Infeasible to verify statically
- Solution: Instrument cvc5 to generate *externally checkable* proofs



Proofs in cvc5

- Covers many parts of the system
- Evaluated on many SMT-LIB theories

  Barbosa et al IJCAR22

Flexible Proof Production in an Industrial-Strength SMT Solver*

Haniel Barbosa[1], Andrew Reynolds[2], Gereon Kremer[3], Hanna Lachnitt[3], Aina Niemetz[3], Andres Nötzli[3], Alex Ozdemir[3], Mathias Preiner[3], Arjun Viswanathan[2], Scott Viteri[3], Yoni Zohar[4], Cesare Tinelli[2], Clark Barrett[3]

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- Highly detailed and complete
- Fine-grained proofs for rewrites, for strings

  Noetzli et al FMCAD22

Reconstructing Fine-Grained Proofs of Complex Rewrites Using a Domain-Specific Language

Andres Nötzli*, Haniel Barbosa‡, Aina Niemetz*, Mathias Preiner*, Andrew Reynolds†, Clark Barrett*, and Cesare Tinelli†

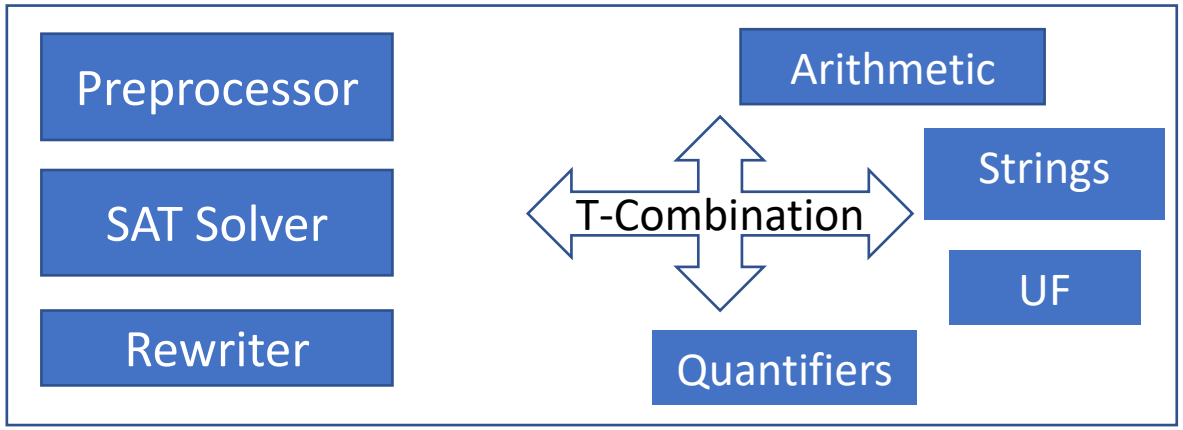
*Stanford University, †The University of Iowa, ‡Universidade Federal de Minas Gerais

Proofs in cvc5: Design Principles

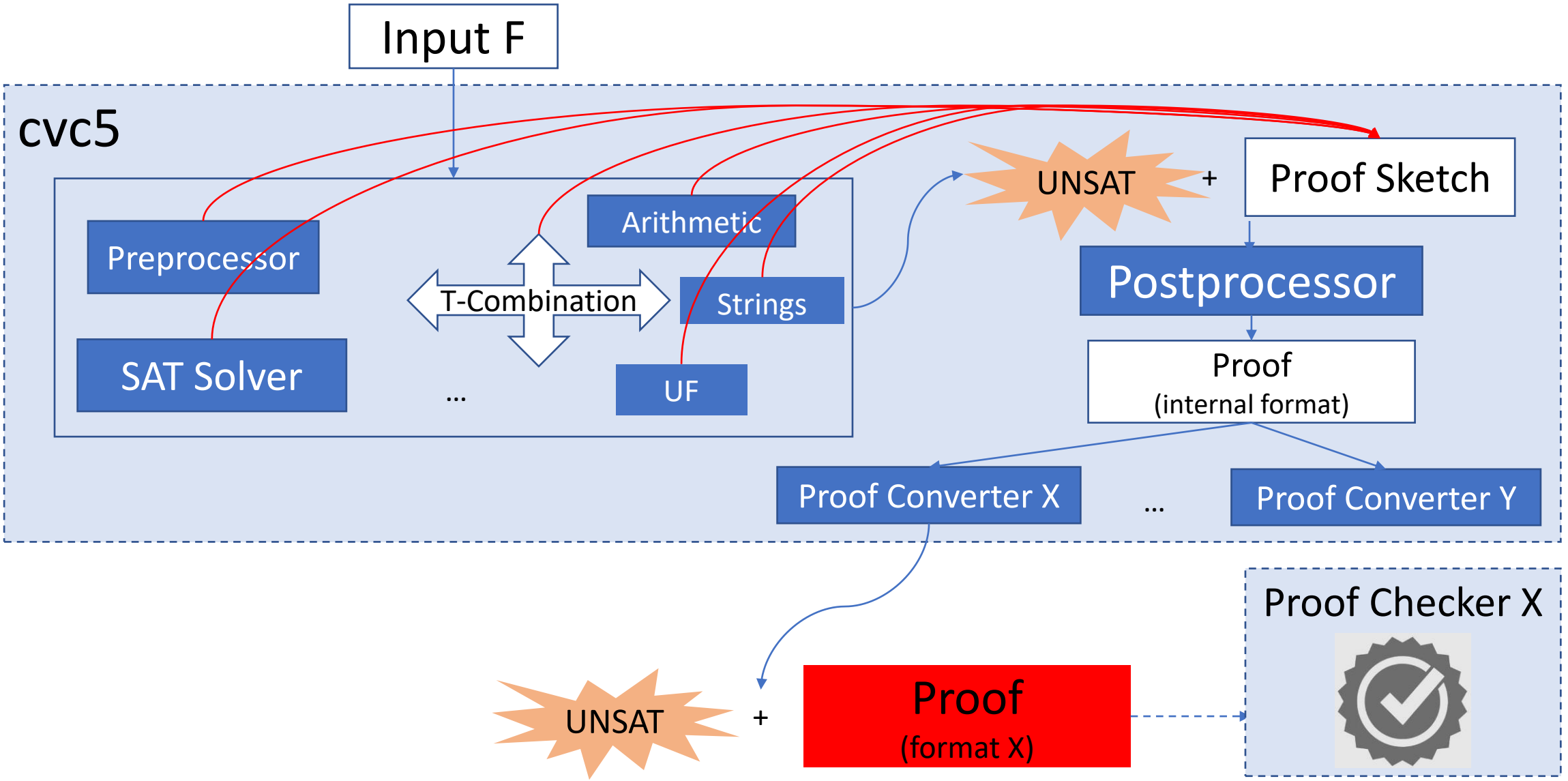
- Flexible
  - Target several backend formats: LFSC, Lean, Alethe, visualization formats
- Also *internally* checkable
  - Use of native proof checker in cvc5 for the purposes of catching errors early
- Provide proofs for all components required for fast solving
  - User should not have to disable features when asking for proofs
- Acceptable performance overhead (~50% performance overhead)
  - Make all optimizations capable of tracking proofs
  - Lazy proof generation

Instrumenting cvc5 for Producing Proofs for Strings

- String solving involves *many parts of the system*:
  - Preprocessing
  - SAT solver (resolution)
  - CNF conversion
  - Theory Combination
  - UF / Congruence closure
  - Linear Arithmetic Solver
  - Rewriting
  - Quantifier instantiation (for reductions)
  - Strings Theory Solver
    - Core calculus (Liang et al CAV 2014)
    - Extended function reductions
    - Regular expression unfolding



Proof Architecture



Future Directions

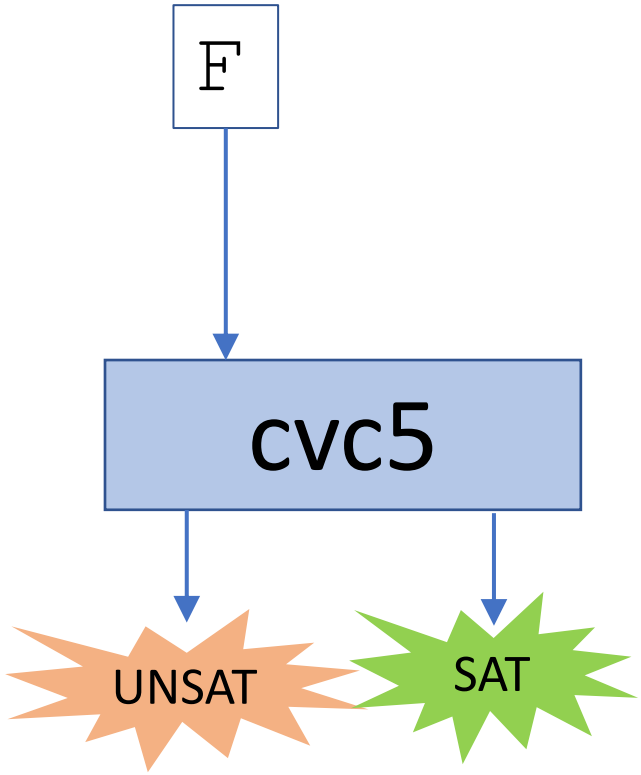
String Solving: Better, Faster

- **Better proofs:**
  - Fine-grained proofs for string rewrites
    - User control over granularity
  - Better integration with external proof checkers
  - Modular extraction of parts of the proof (e.g. SAT skeleton, theory lemmas)

- **Faster solver:**
  - Techniques specialized to constraints of interest to applications
  - More advanced solving architectures

Advanced Architectures in cvc5

- What if we used the CDCL(T) engine as a black box?



Advanced Architectures in cvc5

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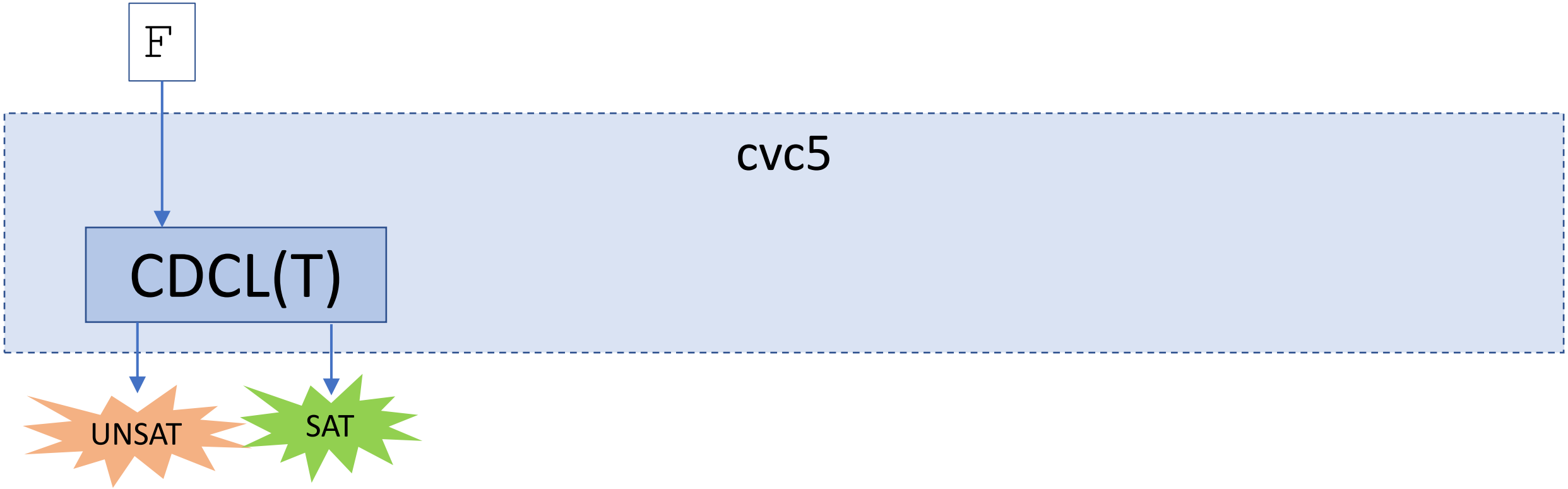
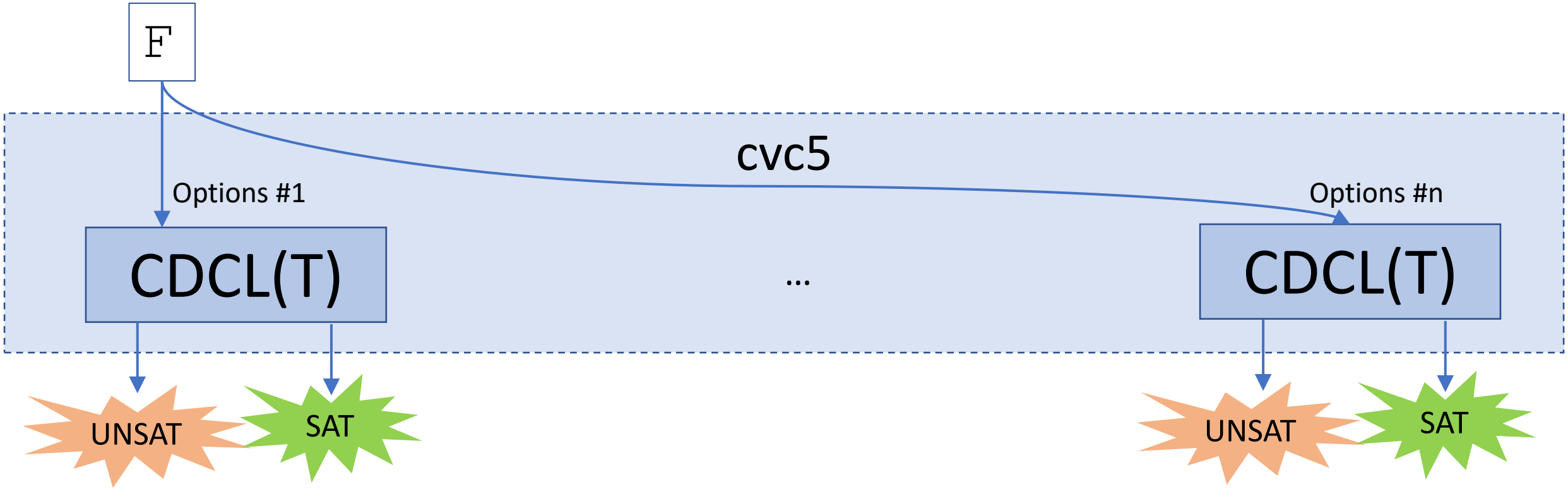
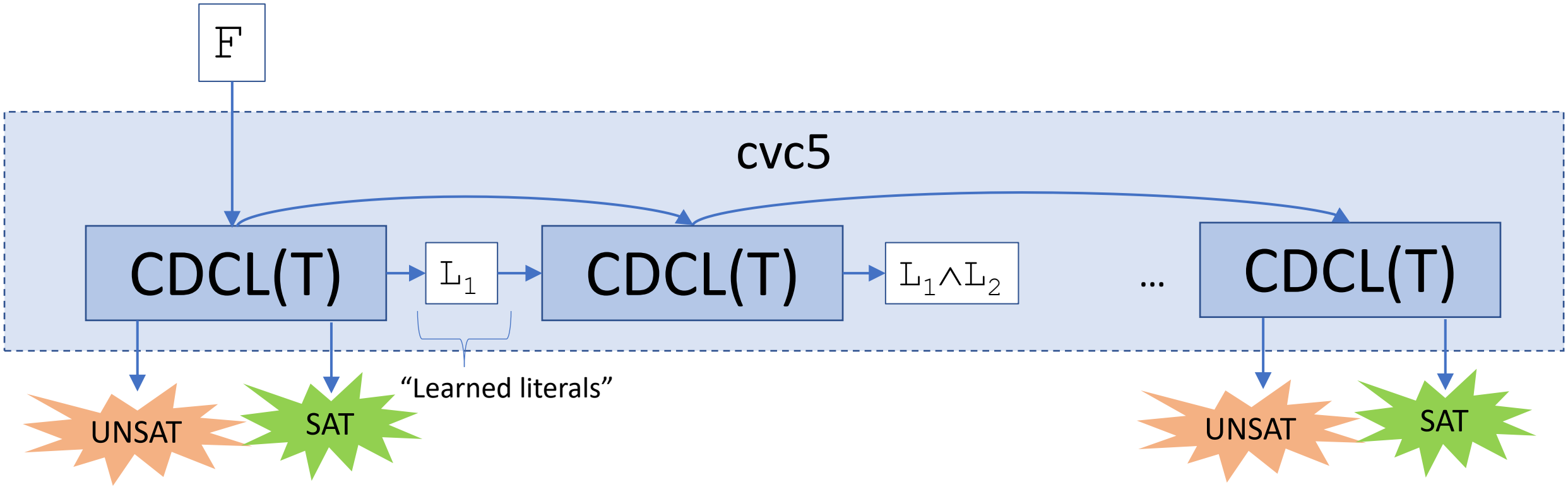


Advanced Architecture: Portfolio



Advanced Architecture: Deep Restarts

- Idea: Restart after learning a set of literals that are implied by F



Deep Restarts

- Given input formula F, a learnable literal l is:
  - Meets some syntactic criteria, e.g. l is a literal from F
  - Is entailed by input, $F \models_T l$

- Strategy to apply deep restarts based on e.g. time threshold
  - Restart, preprocess, solve again

$\Rightarrow$ Preprocessing after learning may make problem significantly easier

Deep Restarts: Possible Variants

- Restart while saving other learned formulas?
  - E.g. theory lemmas based on usefulness criteria
- Maintain SAT solver state on restart?
  - Dynamic mapping between SAT and theory literals
- Save state to disk and restart later?
- Only solve for part of the input formula at a time?

Summary

- SMT solver cvc5 is efficient tool widely used in applications
  - Handles many problem domains
  - State-of-the-art for string solving

- Always looking for new features, faster techniques, increased trust

- Thanks for listening!