# 22C : 231 ($CS$ : 5350 : 0001) Design and Analysis of Algorithms
# Homework 6

The homework has two types of problems – reinforcement problems and regular problems. For the reinforcement problems, we are not concerned with originality in coming up with the solution, but rather with how well you write up the solution. You can get help in coming up with the solution – from friends, online, etc. – but understand the solution and explain it in your own words. For the regular problems, the only type of help you can get is collaboration with classmates, and discussion with the instructor or TA. No record or notes, electronic or written, should be taken from such collaborations. For these problems we do care about originality in coming up with the solution.

The homework is worth 12 points. Each question is worth 2 points. The theme is NP-completeness. The homework is due in class on Tuesday, May 1. (You may have to wait till Tuesday, April 24, before attempting the last two problems.)

## Regular Problems

1. In the decision version of the independent set problem, we are given an undirected graph $G = (V, E)$ and an integer $1 \leq k \leq |V|$, and we want to know if the graph has an independent set of size $k$ or not. Give a direct polynomial time reduction from this problem to the CNF satisfiability problem. By a direct reduction, we mean one that does not use the known fact that CNF satisfiability is NP-complete. **Hint:** Have a variable $x_{ij}$ for each $i \in V$ and $1 \leq j \leq k$. The interpretation is that $x_{ij} = 1$ if $i$ is the $j$-th vertex in an independent set of size $k$. Add clauses that capture independent set constraints.

2. A 2-CNF formula is a CNF formula in which each clause has at most 2 literals. Describe a polynomial time algorithm that decides if a given 2-CNF formula is satisfiable. (I may post a hint about this a little later.)

## Reinforcement Problems

1. Exercise 1 of Chapter 8.

2. Exercise 4 of Chapter 8.

3. Exercise 6 of Chapter 8.

4. Exercise 20 of Chapter 8.