22C : 196 **Computational Geometry**
Homework 4

The problems in this homework are from the text *Computational Geometry: Algorithms and Applications* by de Berg et al., but I have stated such problems to avoid issues that may come up because of using different versions. Each of the following six problems is worth 2 points. They come from Chapters 3, 4, and 5.

1. Give an efficient algorithm to determine whether a polygon $\mathcal{P}$ with n vertices is monotone with respect to some line, not necessarily a horizontal or vertical one. (Exercise from Chapter 3)

2. Suppose that, in the 3-dimensional casting problem, we do not want the object to slide along the facet of the mold when we remove it. How does this affect the geometric problem (computing a point in the intersection of half-planes) that we derived? (Chapter 4)

3. Here is a paranoid algorithm to compute the maximum of a set A of n real numbers:

Algorithm 1 PARANOIDMAXIMUM(A)

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if $|A| = 1$ **then**
    Return the unique element $x \in A$
Pick a random element x from A
$x' \leftarrow$ PARANOIDMAXIMUM($A \setminus \{x\}$)
if $x \leq x'$ **then**
    Return x'
else
    Now we suspect that x is the maximum, but to be absolutely sure, we compare x with the $|A| - 1$ other elements of A.
    Return x

What is the worst case running time of this algorithm? What is the expected running time? Explain. (Chapter 4)

4. A simple polygon $\mathcal{P}$ is called *star-shaped* if it contains a point q such that for any point p in $\mathcal{P}$ the line segment $\overline{pq}$ is contained in $\mathcal{P}$. Give an algorithm whose expected running time is linear to decide whether a simple polygon is star-shaped. (Chapter 4)

5. Let S be a set of n axis-parallel rectangles in the plane. We want to be able to report all rectangles in S that are completely contained in any query (axis-parallel) rectangle $[x : x'] \times [y : y']$. Describe a data structure for this problem that uses $O(n \log^3 n)$ storage and has $O(\log^4 n + k)$ query time, where k is the number of reported answers. Hint: Transform the problem to an orthogonal range searching problem in some higher-dimensional space. (Chapter 5)

6. Let P consist of a set of n polygons in the plane. Again describe a data structure that uses $O(n \log^3 n)$ storage and has $O(\log^4 n + k)$ query time to report all polygons contained in the query rectangle, where k is the number of reported answers.

The homework is to be turned in into the dropbox *Homework4* on ICON. I would prefer if you type in the text, but hand-drawn figures are okay. The homework is due by 11:59 pm on Thursday March 28th. For this homework, I am allowing a maximum of one late day, so that I can post solutions in time for the midterm.

On the question of collaboration and seeking help, I recommend thinking about each problem for 30 minutes first (not counting time spent getting familiar with basic material covered in class). You may collaborate with classmates after that, but definitely avoid looking at completely written solutions of others. Explain the final solution in your own words, and do not turn in a solution that you don't understand.