Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 5

The homework is due in class on Tuesday, April 21. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom.

1. Show that there is a language in SPACE(n^6) that is not in SPACE(n^2). This is a special case of the Space Hierarchy Theorem, which is Theorem 4.8 in the text. Model your proof on that of the Time Hierarchy Theorem (Theorem 3.1). (4 points)

2. Argue that the function $H : \mathbb{N} \to \mathbb{N}$ used in the proof of Ladner's Theorem (Theorem 3.3) is computable in polynomial time. (3 points.) For completeness, here is the definition of H:

   > $H(n)$ is the smallest integer $i < \log\log n$ such that for every $x \in \{0,1\}^*$ with $1 \leq |x| \leq \log n$, M_i outputs $\text{SAT}_H(x)$ within $i|x|^i$ steps. If there is no such number i, let $H(n) = \log\log n$.

   Some notes on the definition:

   (a) $\log\log n$ may not be an integer; if so assume it is rounded up to an integer.

   (b) Define $H(n) = 1$ for $n = 1, 2$; this is needed because $\log 2 = 1$, $\log\log 2 = 0$, etc.

   (c) As we mentioned in class, the definition of H is recursive. In defining $H(n)$ we assume that $H(m)$ has been defined for $m \leq \log n$. Recall that the language SAT_H is the language

   $$\{\psi 0 1^{n^{H(n)}} : \psi \in \text{SAT and } n = |\psi|\}.$$

   (d) M_i is the TM encoded by the binary expansion of i.

   In your solution, explain the algorithm for computing H and do at least part of the accounting for its running time. For example, you could write down a recurrence for the running time, but short stop of solving it if the recurrence is non-standard.

3. Show that the following language SPACE TMSAT is PSPACE-complete. (3 points).

   $$\text{SPACE TMSAT} = \{\langle \alpha, w, 1^n \rangle : M_\alpha \text{ is a TM and it accepts } w \text{ in space } n.\}$$

   Note that $\alpha, w \in \{0,1\}^*$.