Computational Geometry
Homework 3

Several problems in this homework are adapted from the textbook *Computational Geometry: Algorithms and Applications* by de Berg et al., but I have restated the problems for convenience.

1. A simple polygon $\mathcal{P}$ is called *star-shaped* if it contains a point q such that for any point p in $\mathcal{P}$ the line segment $\overline{pq}$ is contained in $\mathcal{P}$. Give an algorithm whose expected running time is linear to decide whether a simple polygon is star-shaped. (Chapter 4)

2. On n parallel railway tracks n trains are going (in the same direction) with constant speeds $v_1, v_2, \ldots, v_n$. At time $t = 0$, the trains are at positions $k_1, k_2, \ldots, k_n$. Give an $O(n \log n)$ algorithm that detects all trains that at some moment in time are leading. To this end, use the algorithm for computing the intersection of half-planes. (Chapter 4)

3. Let S be a set of n axis-parallel rectangles in the plane. We want to be able to report all rectangles in S that are completely contained in any query (axis-parallel) rectangle $[x : x'] \times [y : y']$. Describe a data structure for this problem that uses $O(n \log^3 n)$ storage and has $O(\log^4 n + k)$ query time, where k is the number of reported answers. Hint: Transform the problem to an orthogonal range searching problem in some higher-dimensional space. (Chapter 5)

   For this problem and the next two, it is fine if your space and time bounds are off by a logarithmic factor or two.

4. Let P consist of a set of n polygons in the plane. Describe a data structure that uses $O(n \log^3 n)$ storage and has $O(\log^4 n + k)$ query time to report all polygons contained in the query rectangle, where k is the number of reported answers. (Chapter 5)

5. Let S_1 be a set of n disjoint horizontal line segments and S_2 be a set of m disjoint vertical line segments in the plane. Give a plane-sweep algorithm that counts in $O((m+n)\log(m+n))$ time how many intersections there are between segments in S_1 and segments in S_2. (Chapter 5)

The homework is due via ICON by 11:59 pm on Thursday, March 12. I would prefer if you type-set your submissions, but feel free to include hand-drawn figures.

Please pay attention to the policy on collaboration outlined in the syllabus. In particular, you will get the most out of home work if either you work on your own or collaborate with a fellow student or two in a reasonable way. I will evaluate the originality of your writing while reviewing your work. Feel free to stop by during my walk-in hours if you need help.