

Formal Methods in the Field Air

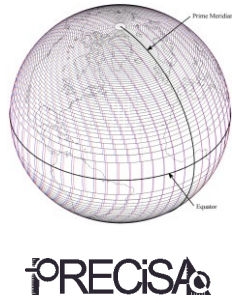
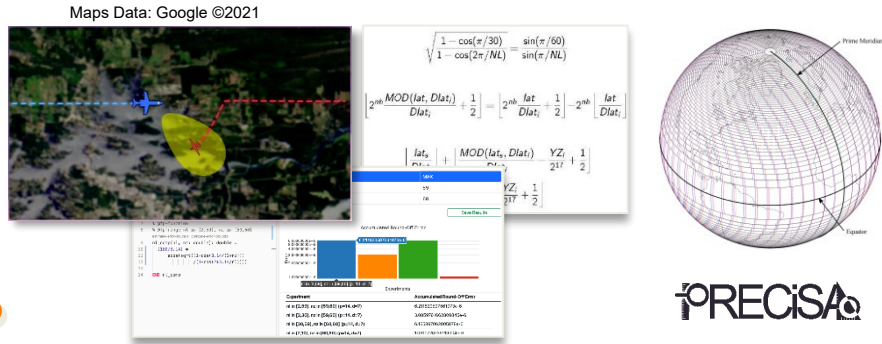
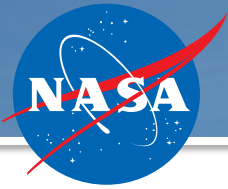
Aaron Dutle*

NASA Langley Research Center

“Formal Methods in the Field” PI meeting,

11-13-2024

*In collaboration with many. Mariano Moscato, César Muñoz, and Laura Titolo most notably.



Over 20 civil aviation standards and guidelines have D320 involvement

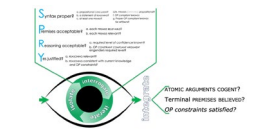
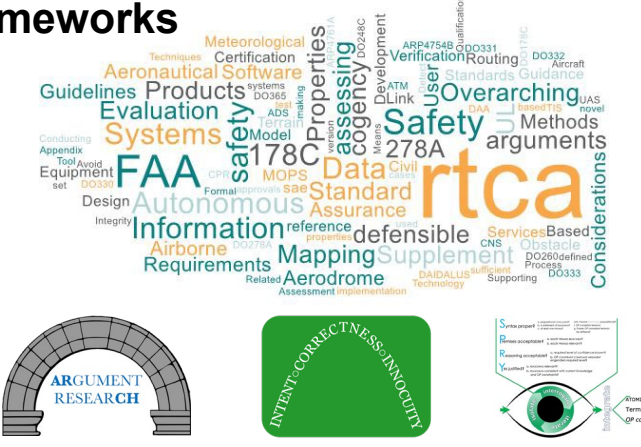
- RTCA DO-365 Detect and Avoid
- RTCA DO-260 ADS-B / TIS-S
- RTCA DO-178C SW Considerations
- RTCA DO-333 Formal Methods Supp.
- RTCA DO-276C Terrain & Obst. data
- RTCA DO-272D Aerodrome Mapping
- ...

Work in progress

- SAE ARP4754B Development A/C Systems
- UL 4600 Standard for Safety for the Evaluation of Autonomous Products
- SAE ARP4761A Guidelines and Methods for Conducting the Safety Assessment Process on Civil Airborne Systems and Equipment
- ...



Safety assessment and assurance frameworks



Application and advancement of formal methods for specifying and verifying correctness and safety properties

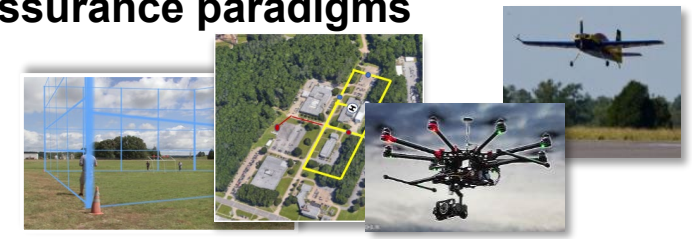
Architectural and runtime V&V for assuring system-level integrity



Highly-assured algorithms for aerospace applications



sUAS flight campaigns to validate environmental assumptions and test blended design and operational assurance paradigms



Maps Data: Google ©2021

$$\frac{\sqrt{1 - \cos(\pi/30)}}{1 - \cos(2\pi/NL)} = \frac{\sin(\pi/60)}{\sin(\pi/NL)}$$

$$\left\lfloor 2^{nb} \frac{\text{MOD}(lat, Dlat_i)}{Dlat_i} + \frac{1}{2} \right\rfloor = \left\lfloor 2^{nb} \frac{lat}{Dlat_i} + \frac{1}{2} \right\rfloor - 2^{nb} \left\lfloor \frac{lat}{Dlat_i} \right\rfloor$$

$$\left\lfloor \frac{lat_s}{Dlat_s} + \frac{\text{MOD}(lat_s, Dlat_s)}{Dlat_s} \frac{YZ_i}{2^{17}} + \frac{1}{2} \right\rfloor$$

- Michael Huerta, Former Administrator, Federal Aviation Administration¹:
*A bedrock principle of aviation is **see and avoid**. And if you don't have a pilot on board the aircraft, you need something that will substitute for that, which will **sense other aircraft**, and we can ensure appropriate levels of safety.*

Code of Federal Regulation requirements for operating in the National Airspace System include:

CFR 91.111: ...not operate so close to another aircraft as to create a **collision hazard**

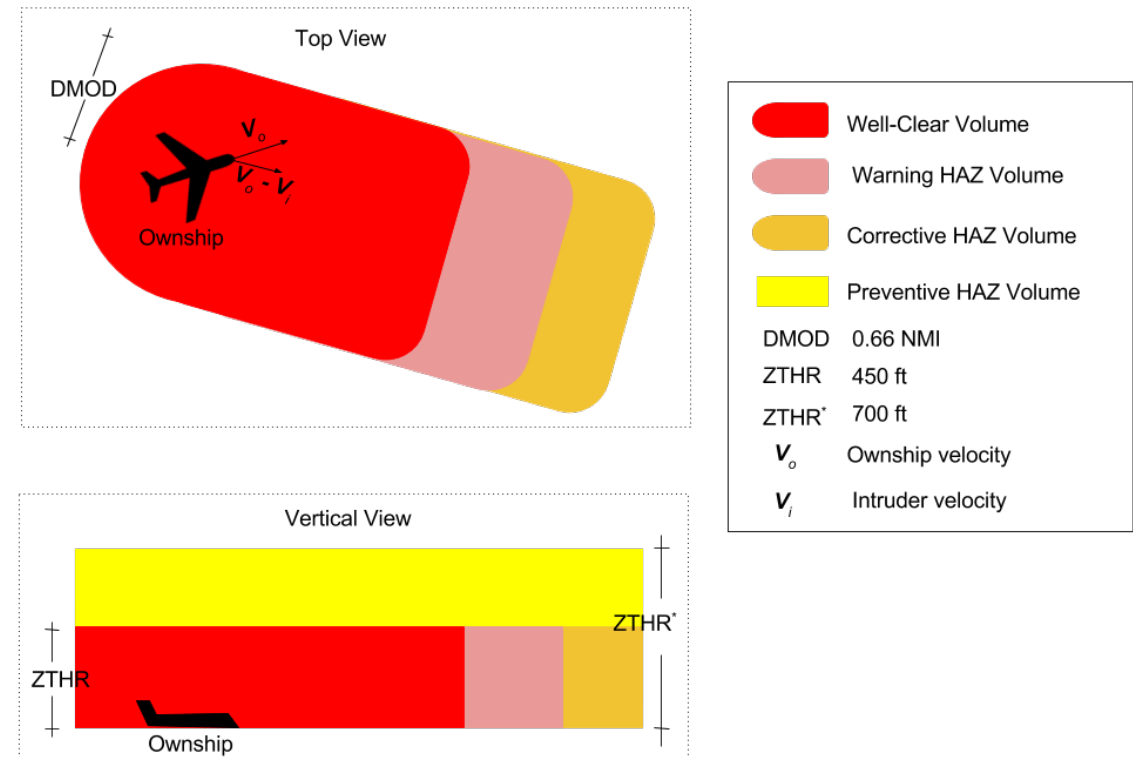
CFR 91.113: Vigilance shall be maintained...so as to **see and avoid** other aircraft...pilots shall alter course to pass **well clear** of other air traffic



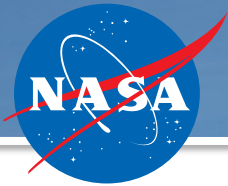
1: <http://www.pbs.org/newshour/bb/drone-industry-grows-faster-flick-joystick-regulation-lags/>

Through the Sense and Avoid Science and Research Panel (SARP), NASA helped develop a geometric **well-clear** definition for uncrewed aircraft.

- Many stakeholders were involved in the panel.
- Several diverse candidates for the well-clear definition were considered.
- At first, experimental evidence, some pilot experiences, and other subjective measures were being considered...
- NASA developed **desired requirements**, and began **formally verifying or disproving** properties of candidate volumes.



Notional depiction of well-clear and alerting volumes



- **Symmetry:** Two aircraft in an encounter calculate the same well-clear status.

Result: Neither aircraft is more burdened than the other.

- **Local Convexity:** In a non-maneuvering encounter, there is at most one time interval with a loss of well-clear.

Result: A conflict or alert will not disappear and then reappear.

- **Extensibility:** Smaller parameter sets create a smaller (nested) volumes.

Result: Alerts will progress monotonically in severity.

- **Convergence:** In a non-maneuvering encounter with a loss of well-clear, the interval includes the time of closest point of approach.

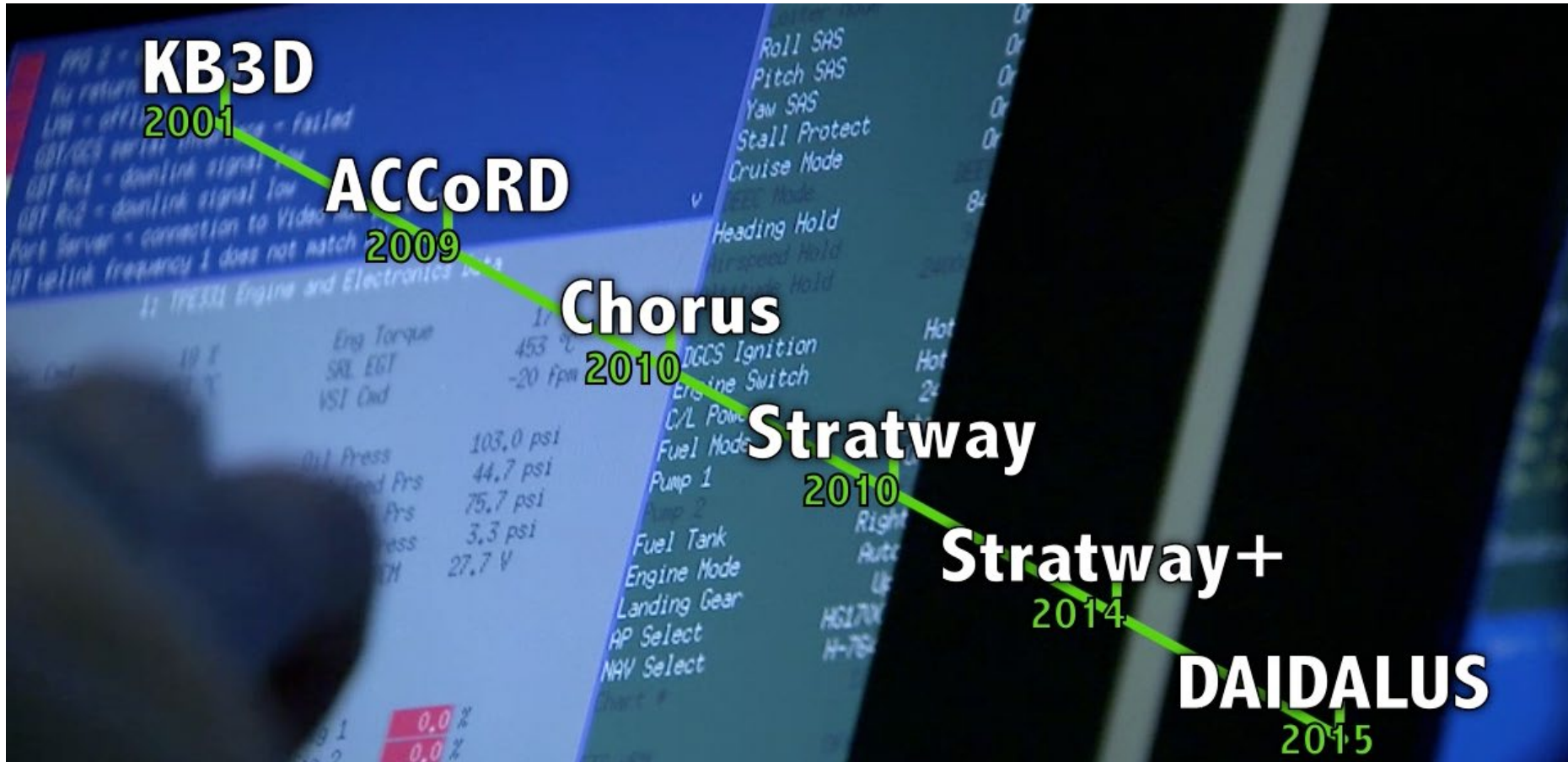
Result: An alert will not disappear before the closest point of approach.

The NASA Formal Methods group **specified** these properties in the Prototype Verification System (PVS), and **verified** that several variants of the eventually chosen candidate satisfied them.

The logo for DAIDALUS features two red, wavy, ribbon-like shapes above the word "DAIDALUS" in a bold, blue, sans-serif font.

DAIDALUS

Detect & **A**void **A**lerting **L**ogic for
Unmanned **S**ystems



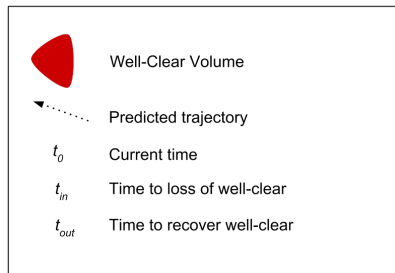
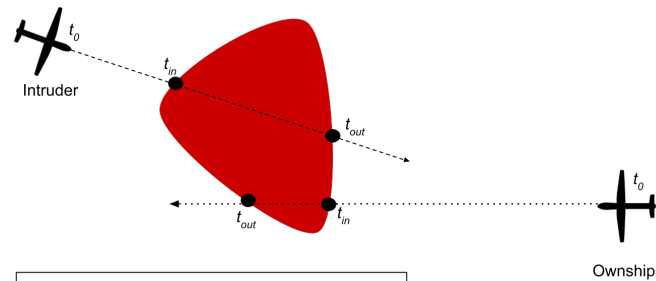
DAIDALUS is the product of years of research on formal V&V methods for the the safe integration of advanced air traffic concepts and algorithms in the National Airspace System.

- DAIDALUS is a reference to the Greek myth.
- Daedalus advised Icarus not to fly too high, or his wings would melt in the sun, and not to fly too low, or his feathers would be soaked.

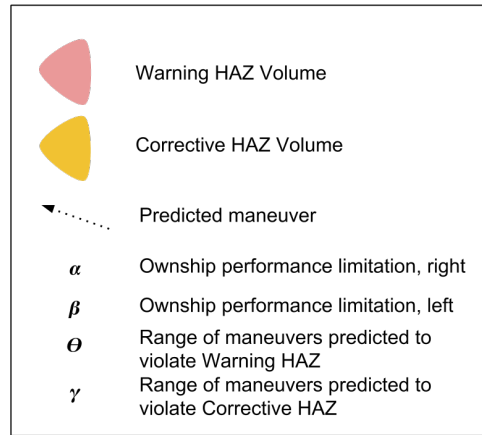
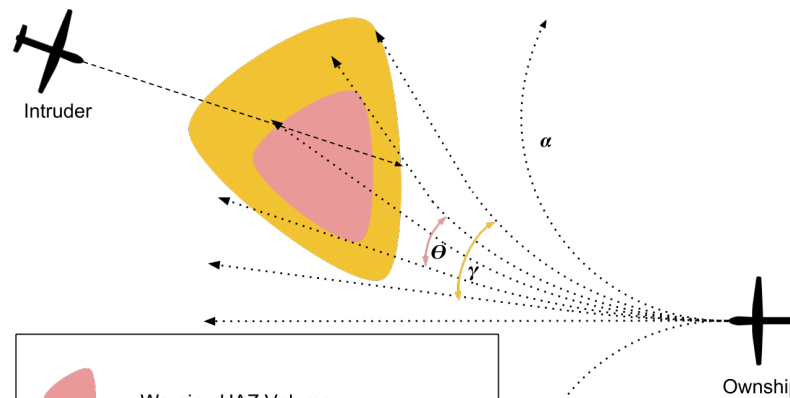


[See page for author](#), Public domain, via Wikimedia Commons

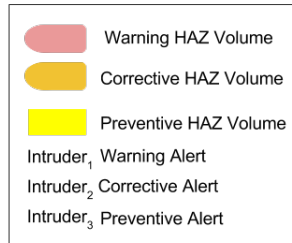
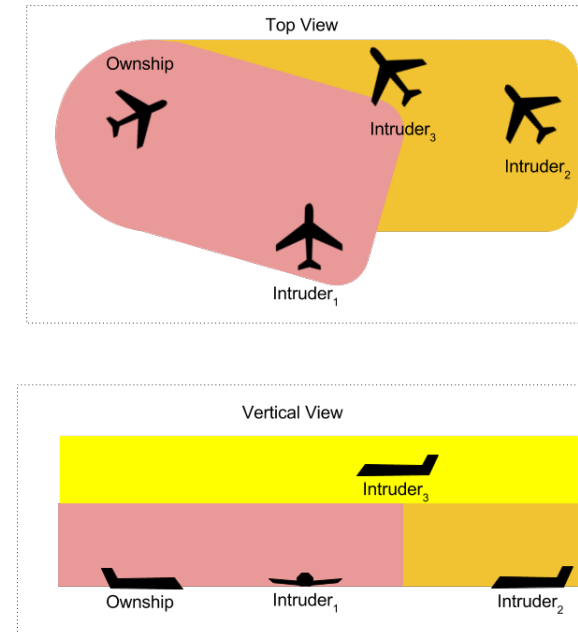
Detection determines the time interval where a loss of well-clear is predicted to occur



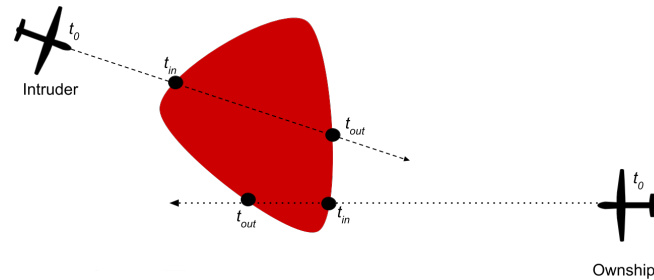
Maneuver Guidance finds ranges of maneuvers that lead to entering some hazard volume



Alerting computes a number indicating the severity of a potential loss of well-clear

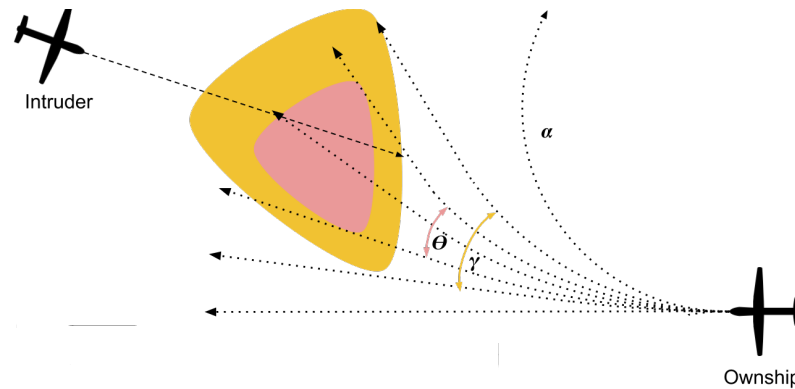


Detection determines the time interval where a loss of well-clear is predicted to occur



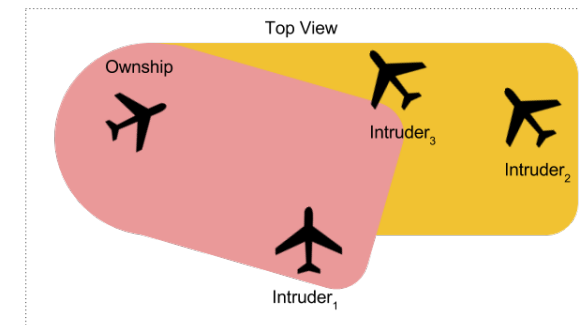
Lemma: (Detection Correctness) Time $t \in [t_{in}, t_{out}]$ if and only if the projected aircraft will have a loss of well-clear in t seconds.

Maneuver Guidance finds ranges of maneuvers that lead to entering some hazard volume



Lemma: (Conflict-free maneuver) If a maneuver is marked NONE, there will be no conflict along the path within the lookahead time.

Alerting computes a number indicating the severity of a potential loss of well-clear



Lemma: (Guidance coordination) An alert is issued if and only if maneuver guidance includes the current trajectory.

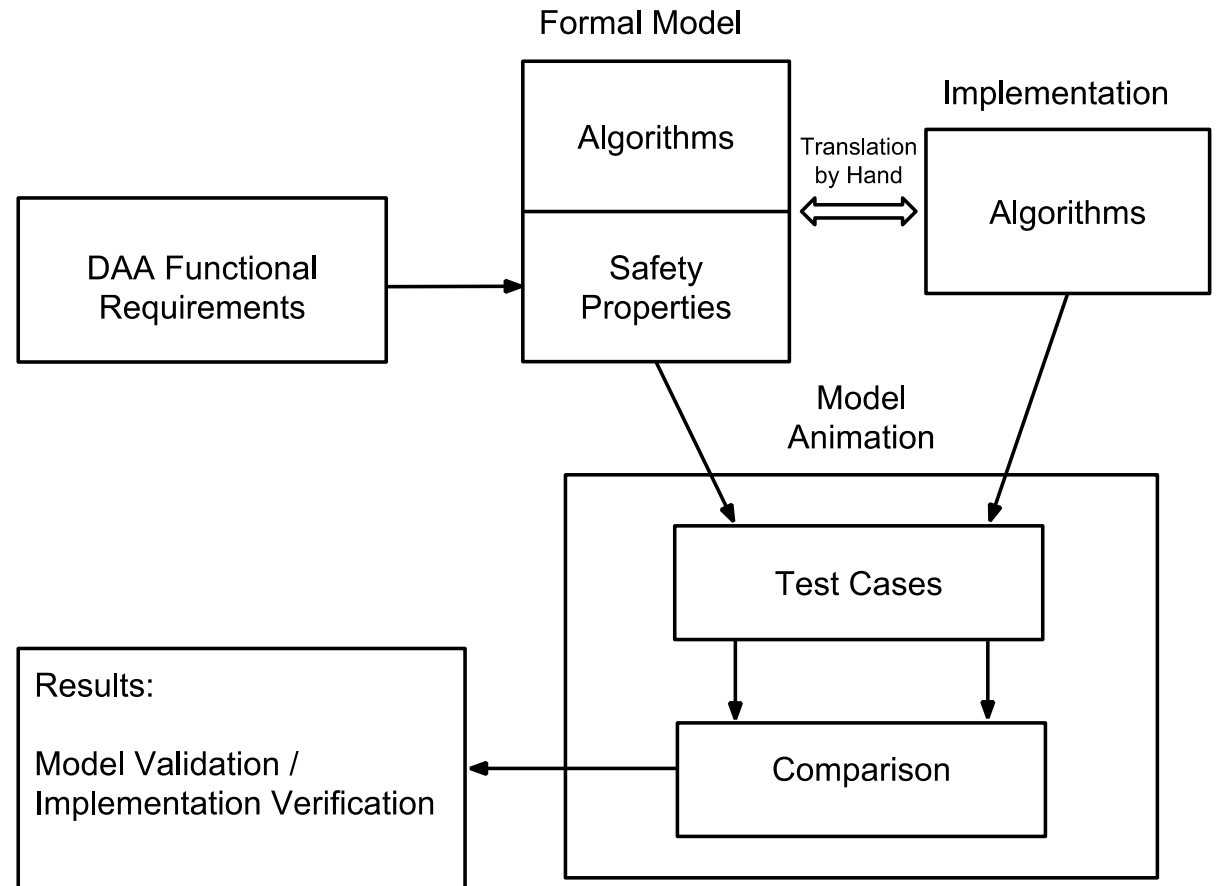
DAIDALUS model in PVS is for verification of core algorithms, not real-world use.

Our user-ready implementations are written in Java and C++.

How do we give assurance that the implementation conforms to the model? With another acronym!

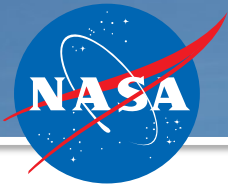
MINERVA

Mirrored Implementation Numerically
Evaluated against Rigorously Verified
Algorithms



- Is the ***reference implementation*** for Detect-and-Avoid in the US standards document maintained by RTCA special committee 228: DO-365 “Minimum Operational Performance Standards (MOPS) for Detect and Avoid (DAA) Systems.”
- Used by partner organizations for test and simulation, and as a benchmark for DAA capability.
- Integrated into currently in-use DAA solutions (e.g., NAVAIR’s Guardian system).
- As the DAA component of ICAROUS and Danti, NASA–developed software.





The RTCA/EuroCAE publishes a standards document, DO-260C/ED-102.

**Minimum Operational Performance Standards for
1090 MHz Extended Squitter
Automatic Dependent Surveillance – Broadcast (ADS-B)
and
Traffic Information Services – Broadcast (TIS-B)**

- Describes a collection of algorithms called **Compact Position Reporting (CPR)** to compress, broadcast and recover the approximate location of an aircraft.
- Reports from pilots and manufacturers say that implementations can be inaccurate.
- FAA official suggested that the Formal Methods team could investigate.

Why don't aircraft just send a GPS location?

- Earth is **BIG**. [citation needed]
- Position needs accuracy to be useful.
- The hardware and software used limit message size to 35 bits.

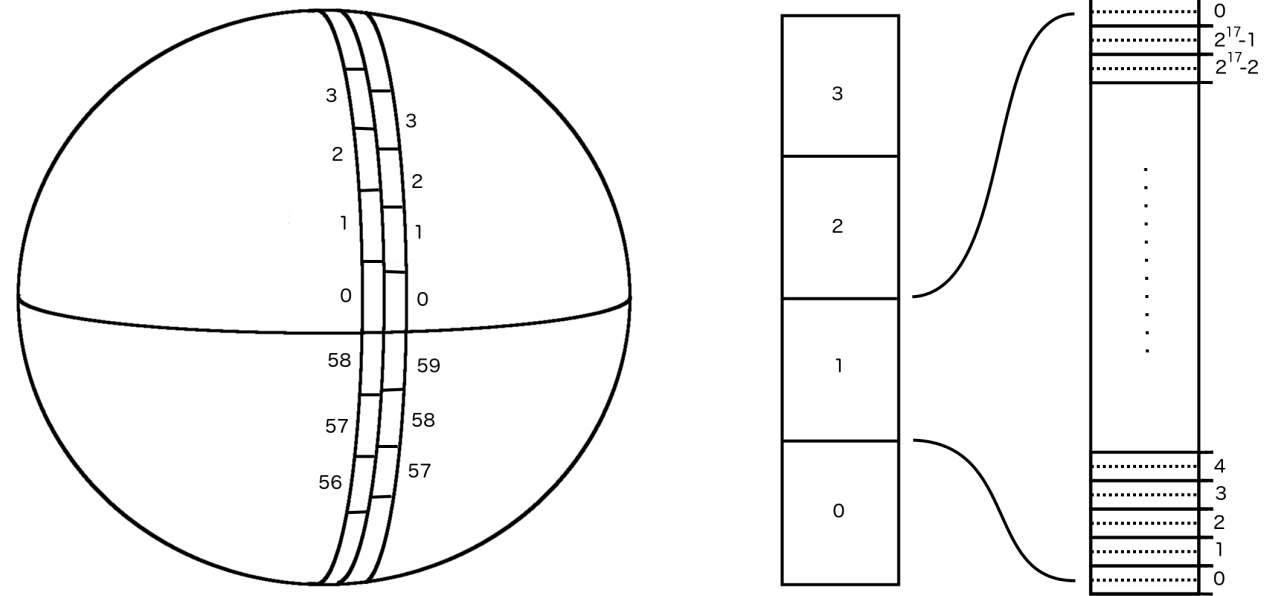


Image Credit: NASA/NOAA/GSFC/Suomi NPP/VIIRS/Norman Kuring
https://www.nasa.gov/sites/default/files/images/618486main_earth_full.jpg

Focus on Latitude:

- Choose a **format** (odd or even)
- Divide latitude into equally sized **zones** (60 for even format, 59 for odd).
- Divide each zone into 2^{17} equally sized **bins**.
- Recovered position is intended to be the centerline of the bin.

Longitude is handled similarly, but the number of zones used decreases toward the poles.

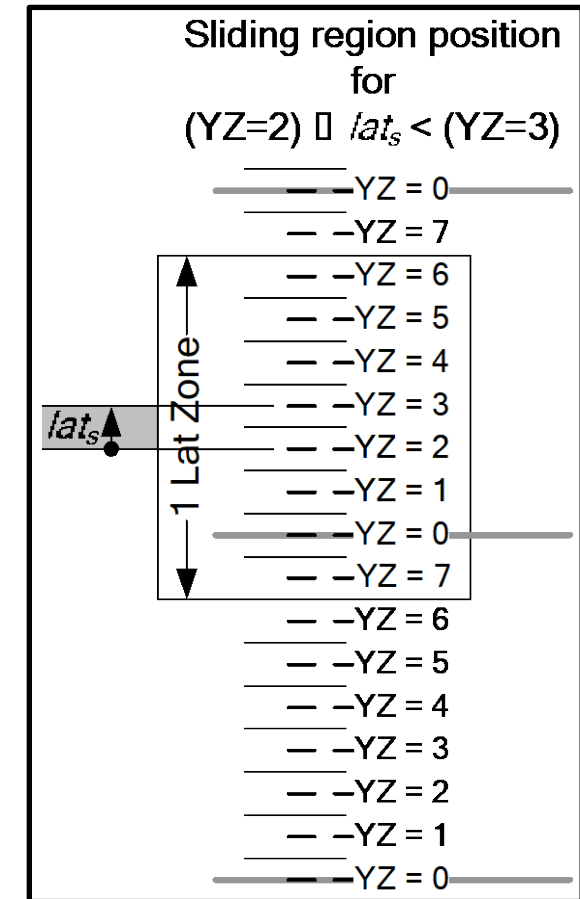


CPR coordinate system for Latitude encoding



Message structure for CPR position

- A global decode is used to establish a first position.
- Global decoding does not compute all positions and compare them. It uses the Chinese Remainder Theorem to compute the zone.
- Local decoding can be used after establishing a position.
- There are distance limits for correct decoding:
 - Local Decoding: The reference point used should be *closer* than **half of a zone length** of the broadcast position.
 - Global Decoding: The pair of positions should be *closer* than **half of a zone offset** (difference between zone sizes).
- These distance limits are converted to time limits via maximum velocity assumptions.



A "one zone" decoding limit around a reference position for local decoding.

Analysis of the algorithm:

- Created a formal model of the algorithm in PVS (Prototype Verification System).
- Discovered incorrect requirements for decoding, and adjusted the requirements to allow for correct operation.
- Found numerous mathematical simplifications for formulas in the CPR specification, to reduce computational error.

- Computing transition latitudes:

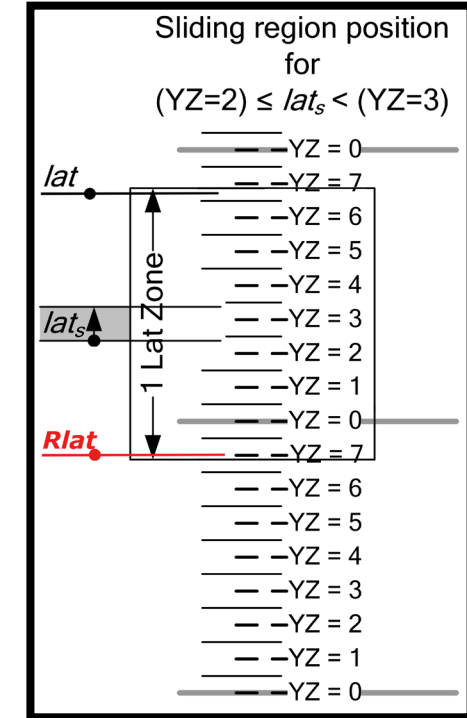
$$\sqrt{\frac{1 - \cos(\pi/30)}{1 - \cos(2\pi/NL)}} = \frac{\sin(\pi/60)}{\sin(\pi/NL)}$$

- During encoding:

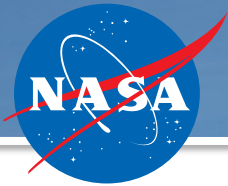
$$\frac{MOD(A, B)}{B} = \frac{A}{B} - \left\lfloor \frac{A}{B} \right\rfloor$$

- During Local Decoding:

$$\left\lfloor \frac{lat_s}{Dlat_i} \right\rfloor + \left\lfloor \frac{MOD(lat_s, Dlat_i)}{Dlat_i} - \frac{YZ_i}{2^{17}} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{lat_s}{Dlat_i} - \frac{YZ_i}{2^{17}} + \frac{1}{2} \right\rfloor$$



Previous CPR requirements could decode to an incorrect “zone”, resulting in aircraft being reported hundreds of miles from their actual position.



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$$\sqrt{\frac{1 - \cos}{1 + \cos}}$$

- During encoding:

MOD

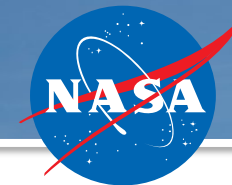
- During Local Decoding:

$$\left\lfloor \frac{lat_s}{Dlat_i} \right\rfloor + \left\lfloor \frac{MOD(lat_s)}{Dlat_i} \right\rfloor$$

---	YZ = 4.9	
---	YZ = 4.8	
---	YZ = 4.7	
---	YZ = 4.6	
---	YZ = 4.5	
---	YZ = 4.4	
---	YZ = 4.3	
---	YZ = 4.2	
---	YZ = 4.1	
---	YZ = 4.0	
---	YZ = 3.9	
---	YZ = 3.8	
---	YZ = 3.7	
---	YZ = 3.6	
---	YZ = 3.5	
---	YZ = 3.4	
---	YZ = 3.3	
---	YZ = 3.2	
---	YZ = 3.1	
---	YZ = 3.0	

$$+ \frac{1}{2}$$

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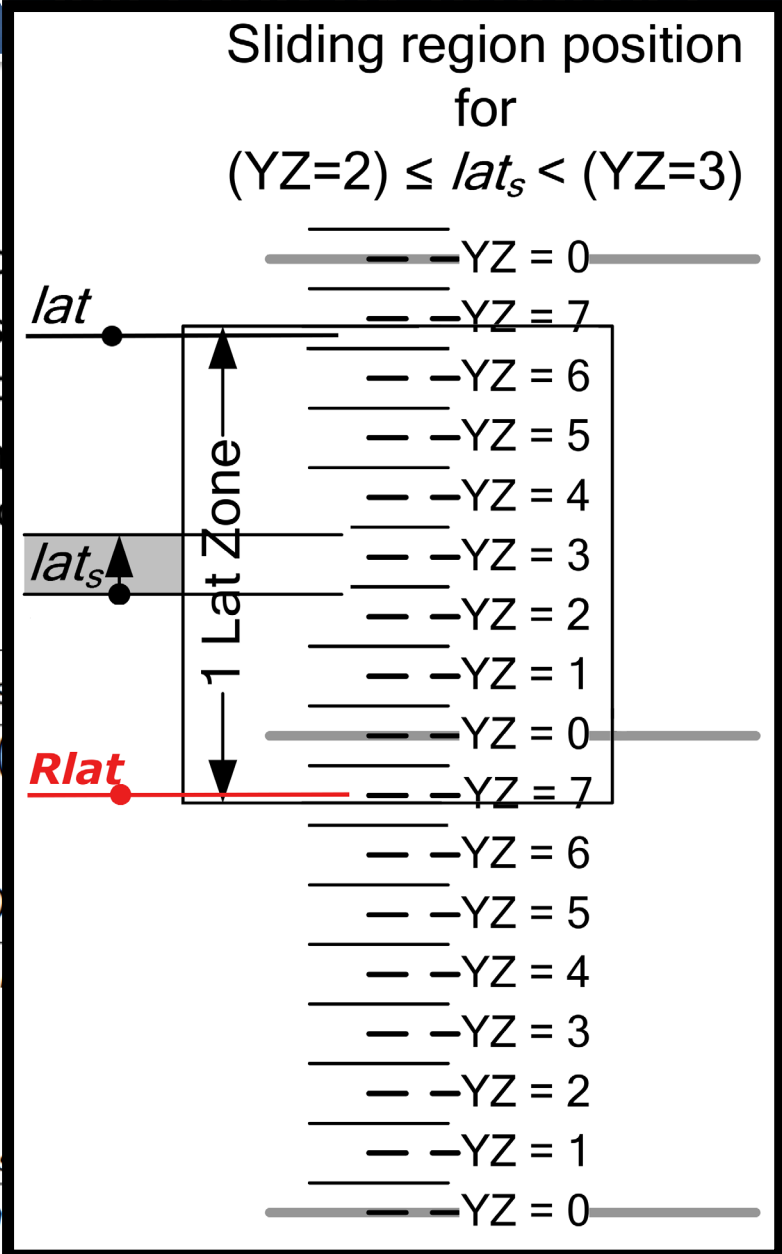
$$\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$$

- During encoding:

MOD

- During Local Decoding:

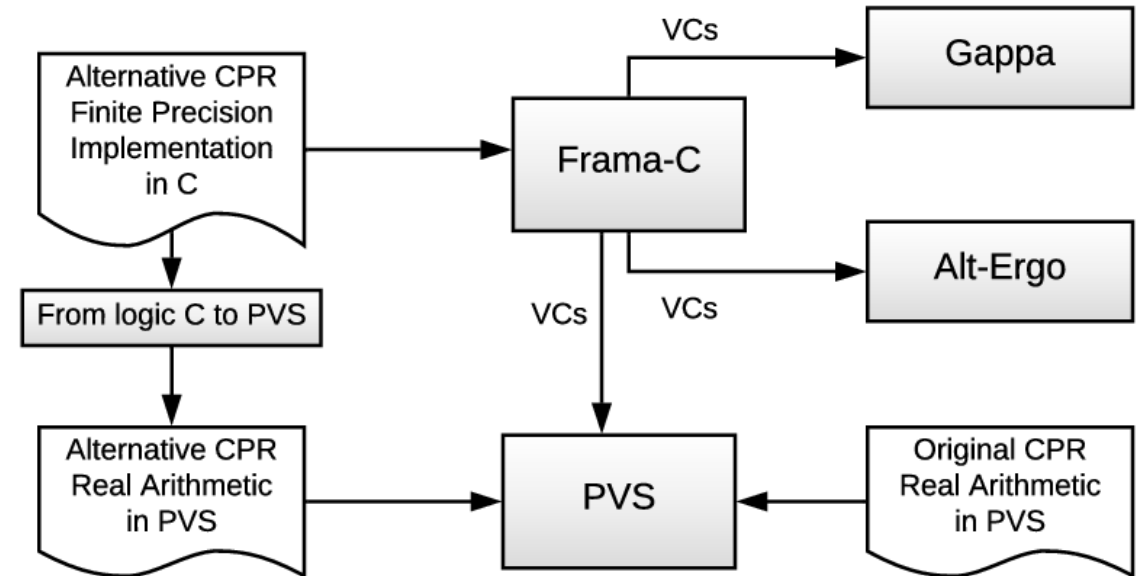
$$\left\lfloor \frac{lat_s}{Dlat_i} \right\rfloor + \left\lfloor \frac{MOD(lat_s, Dlat_i)}{Dlat_i} + \frac{1}{2} \right\rfloor$$



Previous CPR requirements could decode to an incorrect "zone", resulting in aircraft being reported hundreds of miles from their actual position.

Analysis of CPR implementations

- NASA **developed** and **formally verified** versions of CPR in floating point (double precision) and fixed point (single precision).
- Analysis discovered additional **stressing test cases** that would catch some incorrect implementations of CPR.
- Led to development of **PRECiSA** and **REFLOW**, tools for analysis and rewriting of floating point programs.



The formal verification of CPR implementations involved the use and interaction of several formal methods tools.



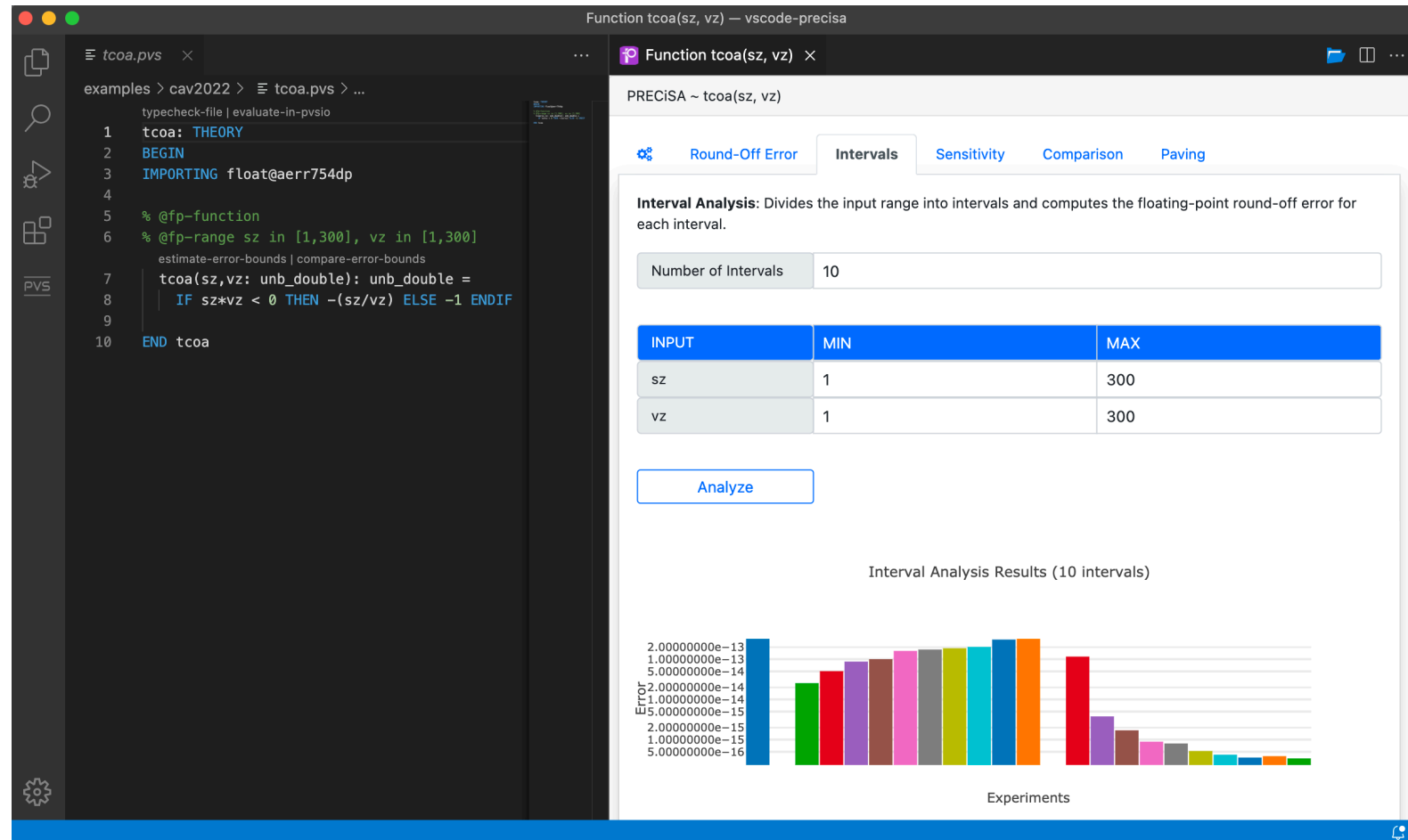
Floating-point Analysis:

- Bounds accumulated error.
- Identifies unstable conditionals.
- Produces verifiable certificates.
- VScode interface for experimentation

REFLOW

Program Transformation:

- Creates a guard-stable version of a program in C.
- Automatically verifies the correctness of the implementation.



Function tcoa(sz, vz) — vscode-precisa

```
tcoa.pvs
typecheck-file | evaluate-in-pvsio
1 tcoa: THEORY
2 BEGIN
3 IMPORTING float@aerr754dp
4
5 % @fp-function
6 % @fp-range sz in [1,300], vz in [1,300]
   estimate-error-bounds | compare-error-bounds
7   tcoa(sz,vz: unb_double): unb_double =
8     IF sz*vz < 0 THEN -(sz/vz) ELSE -1 ENDIF
9
10 END tcoa
```

PRECiSA ~ tcoa(sz, vz)

Round-Off Error Intervals Sensitivity Comparison Paving

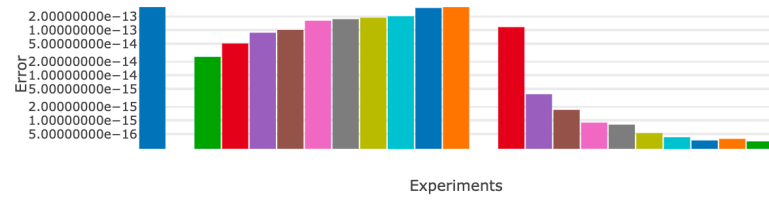
Interval Analysis: Divides the input range into intervals and computes the floating-point round-off error for each interval.

Number of Intervals 10

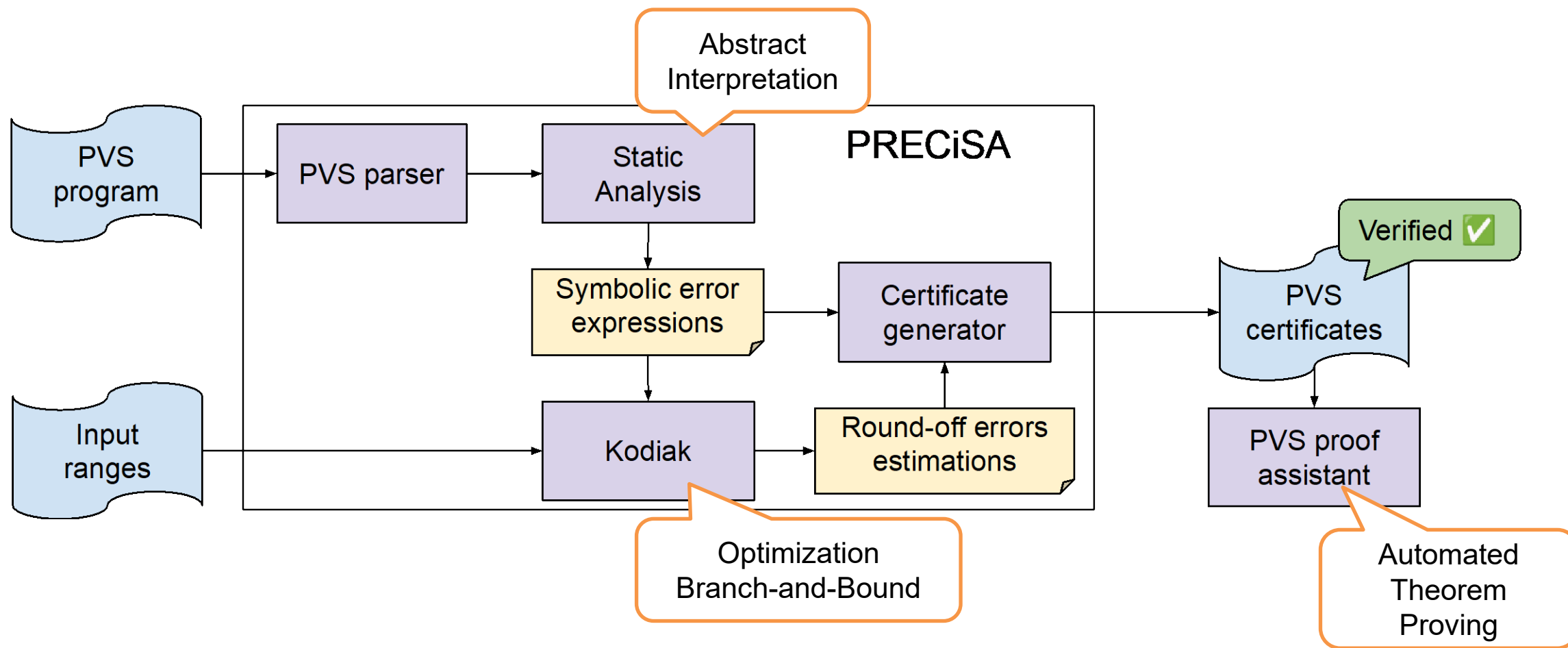
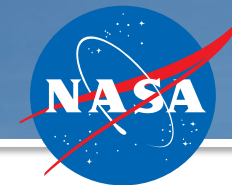
INPUT	MIN	MAX
sz	1	300
vz	1	300

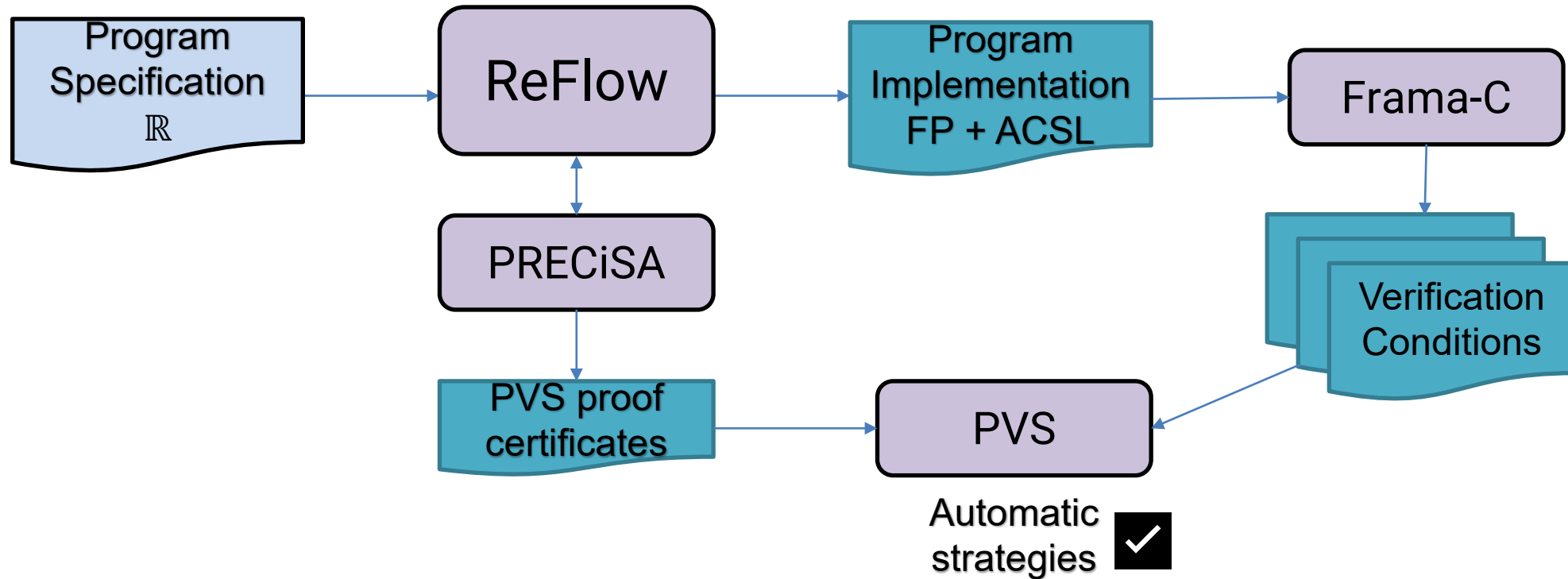
Analyze

Interval Analysis Results (10 intervals)



A view of PRECiSA in VScode allowing visual analysis of the accumulated error over a range of values.





ReFlow takes a real-number specification and generates

- Annotated floating-point C code.
- Proof certificates that verify bounds on error without user interaction.
- Ongoing integration with Herbie will rewriting the original program to lower numerical error.



Ongoing development and enhancements to Formal Methods tools.

- PVS libraries and tools
- Floating point analysis
- Runtime assurance
- ...

Assuring aerospace applications that are safety- or mission-critical.

- Advanced Air Mobility
- Uncrewed Aircraft Systems
- Air Traffic Management

For more information

Formal Methods as NASA:

<https://shemesh.larc.nasa.gov/fm/>

DAIDALUS

<https://shemesh.larc.nasa.gov/fm/DAIDALUS/>

CPR

<https://shemesh.larc.nasa.gov/fm/CPR/>

NASA PVS libraries

<https://shemesh.larc.nasa.gov/fm/pvs/PVS-library/>

PRECiSA and REFLOW

<https://shemesh.larc.nasa.gov/fm/PRECiSA/>

Come work with us!

<https://intern.nasa.gov/>

A Shameless Plug



Submit your paper!

<https://shemesh.larc.nasa.gov/nfm2025/>

For more information

Formal Methods as NASA:

<https://shemesh.larc.nasa.gov/fm/>

DAIDALUS

<https://shemesh.larc.nasa.gov/fm/DAIDALUS/>

CPR

<https://shemesh.larc.nasa.gov/fm/CPR/>

NASA PVS libraries

<https://shemesh.larc.nasa.gov/fm/pvs/PVS-library/>

PRECiSA and REFLOW

<https://shemesh.larc.nasa.gov/fm/PRECiSA/>

Come work with us!

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